What Can We Do for You?
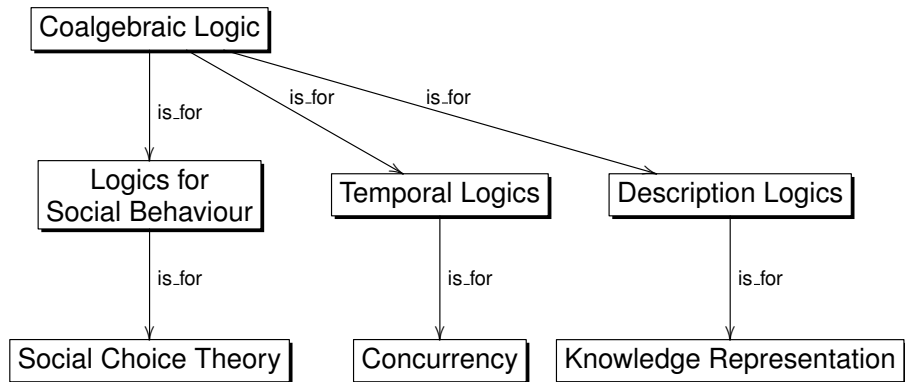## Coalgebraic Logics for Social Behaviour

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There's a tiresome young man in Bay Shore.
When his fiancee cried, 'I adore
The beautiful sea',
He replied, 'I agree,
It's pretty, but what is it for?

Morris Bishop

Being for Something

Coalgebraic Logic

- ► Generic framework for logics with modalities, e.g.
  - ► (Relational)
  - ► Neighbourhood-based
  - ► Preferential
  - ► Probabilistic / weighted
  - ► Game-based
- ► Parametrized by
  - ► Syntax: Choose your operators
  - ► Semantics:
    - ► Pick your system type (coalgebras for a functor)
    - ► Interpret your operators
  - ► Proof theory

Example: Good Old Modal Logic

- Choose your operators: $\Box$, e.g. 'it is obligatory that'

- Pick your system type (i.e. functor):

  Powerset $\mathcal{P}$

  Coalgebras $X \to \mathcal{P}(X)$ are Kripke frames

- Interpret your operators:

$$\llbracket \Box \rrbracket_X(A) = \{B \in \mathcal{P}(X) \mid B \subseteq A\}$$

$$\Box\,\text{stay} \land \Box\text{go} \to \Box\text{die}$$

Example: Neighbourhoods

- Choose your operators: $\Box$, e.g. 'it is desirable that'

- Pick a system type:

$$\mathcal{N}(X) = 2^{(2^X)}$$

  Coalgebras $X \to \mathcal{N}(X)$ = neighbourhood frames

- Interpret your operators:

$$[\![\Box]\!]_X(B) = \{\mathfrak{A} \in 2^{(2^X)} \mid B \in \mathfrak{A}\}$$

$$\Box\,\text{haveCake} \wedge \Box\,\text{eatCake} \not\to \Box\,\text{die}$$

Example: Coalition Logic

- Choose your operators: $[C]$ 'Coalition $C \subseteq N = \{1, \ldots, n\}$ can force'
- Pick a system type (a functor):

$$F(X) = \left\{ (k_1, \ldots, k_n, f) \mid f : \left(\prod_{i \in N} \{1, \ldots, k_i\} \right) \to X \right\}.$$

  Coalgebras $X \to FX$ = game frames

- Interpret your operators:

$$[\![[C]]\!]_X(A) = \{ f \in F(X) \mid \exists \sigma_C. \forall \sigma_{N-C}. f(\sigma_C, \sigma_{N-C}) \in A \}$$

- Magical tableau rules (for $C_i \subseteq D$ pairwise disjoint):

$$A_{km} \quad \frac{[C_1]a_1, \ldots, [C_k]a_k, \langle D \rangle b, \langle N \rangle c_1, \ldots, \langle N \rangle c_m}{a_1, \ldots, a_k, b, c_1, \ldots, c_m} \qquad A'_k \quad \frac{[C_1]a_1, \ldots, [C_k]a_k}{a_1, \ldots, a_k}$$

Example: Probabilistic Belief

▶ Pick your operators: L_p^i 'Agent i believes with certainty at least p'
▶ Pick your system type:

$$\mathcal{D}(X) = \text{Discrete probability measures on } X$$
$$F(X) = \mathcal{D}(X)^I$$

Coalgebras $X \to F(X)$ are type spaces.

▶ Interpret your operators:

$$[\![L_p^i]\!]_X(A) = \{(\mu_i) \in \mathcal{D}(X)^I \mid \mu_i(A) \geq p\}$$

$$L_{1/2}^{\text{Yellen}} \, L_{2/3}^{\text{Draghi}} \, \text{imminentDeflation}$$

Example: Fuzzy Modal Logic

- Pick your operators: $\Diamond$, e.g. likes

- Pick your system type:

$$\mathcal{P}_{\mathsf{fuzzy}}(X) = X \to [0,1].$$

Coalgebras $X \to \mathcal{P}_{\mathsf{fuzzy}}(X)$ = fuzzy Kripke frames

- Interpret your operators:

$$[\![\Diamond]\!]_X(A : X \to [0,1])(B) = \bigvee_{x \in X} A(x) \otimes B(x).$$

$\Diamond\Diamond$ HeavyMetalGuitarist

Orthogonal Features

- ► **Nominals**: names for individuals

- ► **Fixed points:** temporal operators / common knowledge

- ► **Updates:** E.g. public announcement

- ► **First order quantifiers**

$$\exists y (x (\lceil y : \text{fb\_friend}(x, y) \rceil > \lceil z : z = y \rceil))$$

- ► **Monadic second order quantifiers**

Results

- Completeness of Hilbert/Sequent style systems

- Cut elimination

- Upper Complexity Bounds

    - PSPACE for next-step logics

    - EXPTIME for fixpoint logics / global assumptions

    - P for some conjunctive logics

- Compositionality

► Which logics would social choice theory be interested in?

► What problems would one solve using them?

► What questions would one ask about them?