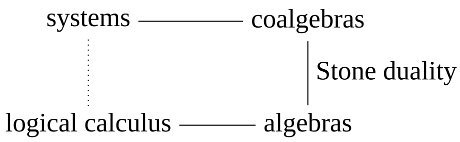
Part 2: Description of Proposed Research

2.1 Introduction and Summary

Transition systems pervade much of computer science. This projects aims at a general theory of specification languages for transition systems. More specifically, transition systems will be generalised to coalgebras as in Rutten [42]. Specification languages together with their proof systems, in the following called (logical or modal) calculi, will be presented by the associated classes of algebras (eg propositional logic by Boolean algebras or intuitionistic logic by Heyting algebras). Stone duality (Johnstone [22]) will be used to give a coalgebraic semantics for the logics represented by algebras.



Stone duality has been used in the ground breaking work of Jónnson and Tarski [23] and Goldblatt [18] in modal logic and Abramsky [2] in computer science. As these works witness, Stone duality is a fundamental tool and it should underly how we think of logics for transition systems.

The proposal The coalgebraic approach to systems allows us to formalise the notion of a type of systems as a functor F. The generality of coalgebras resides in the possibility to build into F many different features like input, output, choice, nondeterminism, probability distributions, etc. The aim of this proposal is to develop the theory of logics for coalgebras not separately for each of these features but parametrically in F. This involves the following issues.

1. Associate to any type F (possibly satisfying some mild conditions to be determined) a corresponding modal logic such that the logic is sound and complete and the semantics fully abstract.

2. Use the parametricity in the type F to describe modularity principles allowing to build complex proof systems from simpler ones. Show how this modularity is useful for the specification and verification of systems.

3. Develop a coalgebraic model theory of modal logic that is parametric in F.

4. Applying the general theory (which will be developed for the items above), it will typically be the case that one wants to relate a set-based semantics to a logic that has a topology-based Stone dual. Thus, the relationship of topologically-based structures and set-based structures will have to be investigated.

Impact The theory of coalgebras set up in [42] is parametric in the type functor F. One of the significant contributions of this project will be to extend this to logics for coalgebras. Indeed, in contrast to other approaches discussed in the next section, we will develop the theory of logics for coalgebras uniformly for all functors F, possibly satisfying some mild conditions to be determined. This will be relevant for applications, as it allows to build up specification languages and proof systems in a modular way.

The project will also be an important contribution to the emerging field of coalgebras and modal logic. In particular, Stone duality will provide a common framework for different previous approaches. Moreover, Stone duality is the natural meeting point for neighbouring disciplines such as semantics of programming languages, domain theory, concurrency, universal algebra, category theory, algebraic logic, and, of course, modal logic (a recent overview, from the point of view of modal logic, of some of these connections is given in Venema [46]). Together with Dr Círstea and Dr Pattinson, I will also organise a first international workshop on this topic in 2005.

Adventure In a wider context, the project concerns the fundamental relationships underlying models of computation as known from concurrency and domain theory on the one hand and modal logics on the other hand. The development of the theory of coalgebras in recent years opens up the possibility of integrating existing insights and to explore new directions. In particular, methods from the model theory of modal logics will be imported into computer science and generalised using coalgebras. The obtained results will impact on program logics as known in domain theory and concurrency and also be exported back to modal logic. The project will thus provide a deeper understanding of systems and their logics as well as of the connections between concurrency, domain theory, modal logic and coalgebras.

2.2 Background

Coalgebras Motivated by Milner's CCS, Aczel [4] introduced the idea of coalgebras for a functor F (on the category of sets) as a generalisation of transition systems. He also made three crucial observations: (1) coalgebras come with a canonical notion of observational or *behavioural equivalence* (induced by the functor F); (2) this notion of behavioural equivalence generalises the notion of *bisimilarity* from computer science and modal logic; (3) any 'domain equation' $X \cong FX$ has a canonical solution (in sets), namely the *final* coalgebra, which is fully abstract wrt behavioural equivalence.

This idea of a type of dynamic systems being represented by a functor F and an individual system being an F-coalgebra, led Rutten [42] to the theory of universal coalgebra which, parameterised by F, applies in a uniform way to a large class of different types of systems. In particular, final semantics and the associated proof principle of coinduction (which are dual to initial algebra semantics and induction) find their natural place here. These ideas have fuelled a large amount of research as witnessed by the proceedings of the annual workshops on Coalgebraic Methods in Computer Science (CMCS 98-04).

Modal Logic Modal calculi were originally developed to axiomatise modalities such as 'necessarily' or 'possibly'. These modal calculi can be given both an algebraic and a relational semantics; the first is based on the so-called Boolean algebras with operators (BAOs) which were introduced by Jónsson and Tarski in the 1950s, a few years before Kripke and others developed the second. The duality theory of algebraic and relational semantics was then developed by Goldblatt and others. The ensuing interaction between calculi, algebraic semantics and relational semantics (or coalgebraic semantics as it can be called today) on the one hand and the advent of applications (arising from the insight that the modality 'necessarily' has similar formal properties as, eg, 'always', 'it is known that', 'provable') in areas like artificial intelligence and economics, concurrency and mobility, and foundations of mathematics on the other hand led to a huge growth of modal logic (see the recent textbooks [9, 12, 26]) and spawned the birth of new disciplines as, eg, multi-agent logic, temporal logic, provability logic.

Coalgebras and Modal Logic As coalgebras provide a general and uniform model for different types of systems, the question arises what appropriate *specification languages for coalgebras* are. The idea to look for modal logics is natural because modal logics have a long tradition as logics for transition systems and they typically respect the above mentioned coalgebraic notion of behavioural equivalence (bisimilarity).

Logics for specifying coalgebras. Research into coalgebras and modal logic started with Moss [32]. The logic of [32] works essentially for any functor F (on the category of sets), but it does not provide the linguistic means to decompose the structure of F which is needed to allow for a flexible specification language. To address this issue, [30, 39] (independently) proposed to restrict attention to specific classes of functors and presented a suitable, but ad hoc, modal logic. This work was generalised by Jacobs [21]. Pattinson showed that these languages with their ad hoc modalities arise from modal operators given by certain natural transformations, called predicate liftings, associated with the functor F. He gives conditions under which *logics given by predicate liftings* are sound and complete [36] and expressive [37].

Coalgebraic modal model theory. From a semantical point of view, modal logic can be considered as dual to equational logic, see [29] (this paper is also an early example of a transfer of results from coalgebras back to modal logic). The results following from this approach work for all functors but the logics need to be strong enough to express all possible behaviours. This needs, in general, infinite conjunctions in the logics. To study finitary logics, Jacobs [21] covers some ground towards a duality for coalgebras/generalised BAOs and Goldblatt [19] develops a notion of ultrapower for coalgebras. Both approaches are restricted again to specific classes of functors.

Applications and Tools The idea of systems as coalgebras and the paradigm of final semantics—together with its associated principles of coinduction—has been applied in such different areas as, for example, automata theory [41], combinatorics [43], control theory [25], process calculi [44, 8, 15, 24], probabilistic transition systems [48, 14], and component-based software development [6, 7]. Modelling classes in object-oriented programming as coalgebras [38, 20] led to new verification tools (LOOP-Tool [45], CCSL [40], CoCasl [34]) which also incorporate reasoning with modal logics based on the research on Coalgebras and Modal Logic described above.

Stone Duality For our purposes Stone duality (Johnstone [22]) can be understood as a general principle providing state-based semantics for calculi represented by algebras. The classical example provided by Stone in 1936 is the duality of Boolean algebras and Stone spaces. Stone spaces are those topological spaces that arise when a Boolean algebra A is represented as an algebra of subsets via an embedding $A \to \mathscr{P}(SA)$ where SA is the set of states used to represent A and $\mathscr{P}$ denotes powerset.

2.3 Proposed Research - Programme and Methodology

The spirit of Rutten's [42] is to develop universal coalgebra uniformly for all functors F. I want to extend this idea to logics for coalgebras. As indicated in the above paragraph on 'Coalgebras and Modal Logic', neither the problem of defining modal logics parametrically in F, nor the problem of a model theory for finitary modal logics for coalgebras have been solved. One of the aims of the proposed research is to show that these difficulties can be overcome using Stone duality.

Stone Duality We think of Stone duality [22, 3] as relating a category of algebras $\mathscr{A}$ representing a propositional logic to a category of topological spaces $\mathscr{X}$ representing the state-based models of the logic. The duality is provided by two contravariant functors P and S,

$$\mathscr{X} \underset{S}{\overset{P}{\rightleftarrows}} \mathscr{A} \, . \tag{1}$$

P maps a space X to the logic of propositions on X and S maps an algebra to its 'canonical model'. Building on this idea, Abramsky [2, 1] has shown how to relate the type constructors of domain theory acting on the semantic (ie topological) side to analogous constructions on the logical (ie algebraic) side, thus providing a logical description of domain theory.

The Proposal - Methodology and Aims In Abramsky's work, a basic Stone duality was extended by 'synchronising' dual constructions on both sides of Diagram (1). This suggests to consider dual functors F on $\mathscr{X}$ and L on $\mathscr{A}$

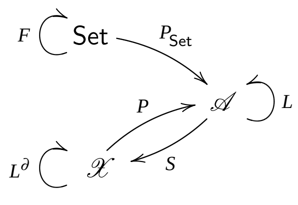
$$F \circlearrowleft \mathscr{X} \underset{S}{\overset{P}{\rightleftarrows}} \mathscr{A} \circlearrowright L \tag{2}$$

where F and L are *dual* if there is an isomorphism $\delta : LP \to PF$ (in the following, we write L^∂, F^∂ for the functors dual to F, L, respectively).[1] In the situation of Diagram (2), dual functors F and L induce a duality between F-coalgebras and L-algebras. In other words, we have *a principle of constructing new Stone-type dualities from simpler ones*. This idea will be applied to the following four issues (see next page for more details).

1. *Associating a modal logic to F.* Given a type F of systems, the initial F^∂-algebra describes an 'abstract' logic for F-coalgebras which is, by duality, sound, complete and expressive in the sense that non-bisimilar states can be distinguished by some formula. What is still lacking on the way to a modal *calculus* for F are the operators to build formulae inductively and the axioms that define derivability, ie an explicit construction of the proof system.

2. *Modular proof systems.* It is common to build complex functors from simple ones using type constructors such as functor composition and (co)product. Describing the duals of these will allow us to build complex proof systems from simpler ones preserving the properties of soundness, completeness, and expressiveness.

3. *The model theory of coalgebras and modal logics.* Assume a type F of (set-based) systems and a (finitary) logic for F-coalgebras described by a functor L on a category of algebras $\mathscr{A}$. Then another relational semantics for the logic is given by the L^∂-coalgebras based on topological spaces. This leads to a modification of Diagram (2)



(3)

where two different semantics, one over Set and one over topological spaces in $\mathscr{X}$, interact. Typically, the calculus corresponding to L will be sound and complete for both semantics, but only the L^∂-semantics will be fully abstract (in other words, only for L^∂-coalgebras the logic will distinguish all bisimilar states). The proposal here is to study the F-semantics for L by studying the relation between L^∂ and F.

4. *Relating set-based and topology-based structures.* The relationship between set-based and topology-based structures (as eg F-coalgebras and L^∂-coalgebras in Diagram (3)) is at the heart of the relationship between models of computation and their logics. For example, domain equations for bisimulation may be solved in the categories of sets [4], SFP-domains [1], and Stone spaces [28]. In each case, the obtained notion of bisimulation is different and the question arises what their relationship is. The aim of this item is to develop a general theory, as already suggested by Abramsky in [1], where these kind of questions can be addressed and answered.

[1]The *dual functor* of $F : \mathscr{X} \to \mathscr{X}$ is $F^{\mathrm{op}} : \mathscr{X}^{\mathrm{op}} \to \mathscr{X}^{\mathrm{op}}$. Since F^∂ is F^{op} up to equivalence $\mathscr{X}^{\mathrm{op}} \simeq \mathscr{A}$, we also call it 'dual of F'.

Preliminary Results (a) The classical duality of modal logic, ie the duality between descriptive general frames and Boolean algebras with operators [18], is a special instance of the duality of F-coalgebras and L-algebras, see [28].
(b) A logic $\mathscr{L}$ given by predicate liftings for F-coalgebras (Pattinson [36]) can be understood as a functor L on boolean algebras (describing modal operators and axioms) and a natural transformation $\delta : LP_{\mathsf{Set}} \to P_{\mathsf{Set}}F$ (describing the semantics, see Diagram (3)). The results of (Pattinson [36, 37]) can be proved using Stone duality, see [27].
(c) Building on the duality of T0-spaces [10], we described the convex compact powerdomain (and its dual) for T0-spaces and showed how to uniformly obtain a number of dualities by restricting to subcategories of T0-spaces such as posets, spectral spaces, SFP-domains, sets, Stone spaces, compact ultrametric spaces, see [11].
(d) *Algebraic logic* [16] associates to each logic a class of algebras and classifies logics wrt algebraic properties. Logics that admit a Stone-type duality are known as self-extensional logics. In Diagram (2) then, $\mathscr{X}$ need not be a category of topological spaces anymore. But it is still possible to describe a powerdomain and its dual, see [31].

Objectives and Detailed Research Programme The first item below will settle questions left open in previous work, whereas 1 to 4 correspond to the aims listed above. 5 describes further natural directions.

0. Continuing **(b)** above, for a functor F and a logic $\mathscr{L}$ given by predicate liftings, study the functor L induced by $\mathscr{L}$. In particular: – Prove that $\mathscr{L}$ is sound and complete if *and only if* δ is injective. Similarly, if F is a functor on Stone spaces, prove that $\mathscr{L}$ is sound, complete, and expressive only if L is dual to F.[2] – Let F be a functor on sets that preserves finite sets. Such a functor extends (from finite sets) to a functor F' on Stone spaces. Prove that $\mathscr{L}$ is sound and complete if and only if F' is L^{∂}.

1. *Associating a modal logic to F.* To obtain a modal calculus for F, is to describe F^{∂} by *generators and relations* [47]. Although it is well-known how to do this in some examples, it is not clear to what extend this can be done in general. When can a category of F^{∂}-algebras be described by a signature and equations.[3] Moreover, to be parametric in F, we want to obtain the equations (and the signature) for F^{∂}-algebras from equations for $\mathscr{A}$ and equations for F^{∂}. We can assume that each $F^{\partial}A$, $A \in \mathscr{A}$, can be described by generators and relations, but, for our purpose, this description need to be uniform in A. Give a precise definition of 'uniform in A' and characterise the functors F^{∂} that have such a description.

2. *Modular proof systems.* To describe the duals of the common type constructors by generators and relations is an application of standard techniques. That soundness, completeness, expressiveness are preserved is a consequence of duality. The main point here is to study different application areas.

3. *The model theory of coalgebras and modal logics.* In the model theory of modal logic [9, 12, 26] a particular instance of Diagram (3) is studied in detail, namely where F is powerset, $\mathscr{A}$ is boolean algebras, L is the functor associated with the standard basic modal logic, and L^{∂} is the Vietoris space on Stone spaces. In the first instance, we keep Boolean algebras but generalise the tools developed in modal logic to arbitrary F-coalgebras. For example, one can transform every F-coalgebra into a L^{∂}-coalgebra in a way that generalises the ultrafilter extension of a Kripke frame.[4] This opens the possibility to prove classic theorems (eg Goldblatt-Thomason) for coalgebras. Similar considerations will lead to coalgebraic analogues of model theoretic concepts such as saturation and bisimulation-somewhere-else.

4. *Relating set-based and topology-based structures.* In order to relate coalgebras over different categories, one has to develop a notion of a functor being 'the same' on *different* categories. One approach, based on ideas of **(c)** above, is to specify a functor on a super-category (such as T0-spaces) which contains the other semantic categories $\mathscr{X}$. Another approach is based on the observation that many of the relevant categories $\mathscr{X}$ arise as different completions of the same small category;[5] this allows to consider functors 'the same' if they agree on the small category and are continuously extended to the large categories $\mathscr{X}$. Analogous investigations on the algebraic side will also be fruitful (transforming Diagram (3) into a square by splitting also the right-hand side). Again, special cases have been investigated in modal logic (and universal algebra/lattice theory) under the name of perfect extension [23] or canonical extension [46]. This will yield another opportunity for a transfer of methods and results.

5. Continuing **(d)** above, I want to characterise those logics that can be given (or can be expanded to logics that can be given) a set-theoretic semantics using Stone duality. Another direction starts from the observation that the research sketched above concerns basic modal logics in the sense of propositional logics extended by (one-step, next-time)-modalities plus axioms to describe the functor F. Once these logics are understood, we will also want to extend these logics by fixed point operators as in the μ-calculus.

---

[2] These will need some assumptions on $\mathscr{L}$. The easier converse was shown in [27].

[3] Even if the category of F^{∂}-algebras has free algebras, the forgetful functor to sets need not be monadic.

[4] This definition of the ultrafilter extension depends on the choice of L. This can be avoided in important cases (including the classical one) where we can define L^{∂} directly in terms of F, for example, if F preserves finite sets.

[5] For example: sets, Stone spaces, compact ultrametric spaces are completions of the category of finite sets; posets, spectral spaces, SFP-domains are completions of the category of finite posets. For SFP-domains and Stone spaces this has been used in [5].

2.4 Relevance to Beneficiaries, Dissemination and Exploitation

Relevance to beneficiaries In general terms, the researchers currently working on coalgebras and modal logic will benefit from the new conceptual foundations arising from an approach based on Stone duality. The coalgebra community (the annual CMCS workshops have approximately 16 accepted papers and more than twice as many participants) will profit from new connections with other disciplines. Researchers interested in specification formalisms for transition systems will be interested in the modular proof systems arising from the approach. Finally, I believe that the neighbouring disciplines such as domain theory, concurrency theory, algebraic logic, and modal logic will profit from the investigations on the relationship of set-based and topology-based structures.

More concretely, my research [30] on specifying coalgebras has already been taken up, developed further and built into the tools developed in Nijmegen (LOOP), Dresden (CCSL) and Bremen (COCASL). Our preliminary study [28] has been followed by Palmigiano [35] and Moss and Viglizzo [33]. Inside the UK, this proposal also provides a conceptual foundation for recent work of Cîrstea [13, 14] (whose proof and language constructors for F-coalgebras are essentially explicit descriptions of dual functors F^{∂}) and sheds new light on Ghani et.al. [17] (whose logics for coalgebras had problems related to the fact that they considered coalgebras over base categories that are locally finitely presentable (*lfp*) whereas the topological base categories considered in this proposal are rather co-lfp but not lfp). I am also cooperating with Baltag (Oxford) and Jung (Birmingham) who do related research on coalgebras and Stone duality, respectively. The connections with algebraic logic are closely related to recent work of Priestley (Oxford). Although the proposed approach is quite different from the former EPSRC-project 'Topological Duality for Modal, Temporal and Program Logics' (Rydeheard, Manchester) we are in contact exploring the possibilities for future cooperations.

Collaborations I am closely collaborating with Bonsangue (Leiden), Venema (Amsterdam) in the Netherlands; Hennicker and Pattinson (Munich, Germany); Rosický (Brno, Czech Republic); Palmigiano (Barcelona, Spain). They work in such different areas as software engineering and formal methods (Bonsangue, Hennicker), modal logic (Venema), category theory (Rosický), algebraic logic (Palmigiano), and all of them profit from the ongoing research on coalgebras and modal logic.

Dissemination and exploitation I will continue my activities which include the following. Publishing in leading journals and conferences. Participating in the annual workshops on Coalgebraic Methods in Computer Science. Giving courses on summer schools and, in particular, the annual spring school on foundations of computer science organised by the Midlands Graduate School (as I did already twice to introduce an international audience of PhD students to 'Coalgebras and Modal Logic' and 'Stone Duality').

2.5 Justification of Resources, Additional PhD Student, Management

Justification of resources, additional PhD student Since the project draws on ideas from different areas such as universal algebra, category theory, semantics of programming languages and modal logic an RA for two years will be required. The department will add a PhD student (GTA, 4 years) who will have time to get acquainted with the area and then continue the work of the RA for two more years. Travelling money is needed to maintain collaboration in the UK and internationally (I have estimated 2 conferences, 4 international and 6 UK travels per year for the RA and for myself). Specialist books from different disciplines will be essential. The technical staff will assist with equipment and system maintenance.

Feasibility, Management Feasibility and adventure have been carefully balanced. Feasibility is demonstrated by the preliminary results and the fact that items 0, 1, 2 and parts of 3 (see Objectives and Detailed Research Programme) consist of concrete problems which I expect to be solved. The insights gained will prove useful in tackling the more adventurous item 4. In particular, the generalisation to coalgebras of methods from the model theory of modal logic following from 3 will form the basis for investigating the general problem of how set-based and topologically-based structures interact in 4. Item 5 is intended for possible extensions but could also be approached independently.

A diagrammatic workplan including the PhD student is presented in an appendix. The variety contained in items 0 to 5 of the Detailed Research Programme will allow to adapt the workplan to the background and interests of the RA and the PhD Student. This will allow us to profit from their individual strengths to the fullest. Regular meetings with experts in the neighbouring disciplines will form an essential part of the project.

References for Part 2

[1] S. Abramsky. A domain equation for bisimulation. *Information and Computation*, 92, 1991.

[2] S. Abramsky. Domain theory in logical form. *Annals of Pure and Applied Logic*, 51, 1991.

[3] S. Abramsky and A. Jung. Domain theory. In *Handbook of Logic in Computer Science*. OUP, 1994.

[4] P. Aczel. *Non-Well-Founded Sets*. CSLI, Stanford, 1988.

[5] F. Alessi, P. Baldan, and F. Honsell. Partializing Stone Spaces Using SFP Domains (extended abstract). In *CAAP '97*, LNCS 1158, 1997.

[6] F. Arbab and J. Rutten. A coinductive calculus of component connectors. In *WADT'02*, LNCS 2755, 2003.

[7] L. Barbosa. Towards a calculus of software components. *J. Univ. Comp. Sci.*, 9, 2003.

[8] F. Bartels. Generalised coinduction. *Math. Structures Comput. Sci.*, 13, 2003.

[9] P. Blackburn, M. de Rijke, and Y. Venema. *Modal Logic*. CSLI, 2001.

[10] M. Bonsangue, B. Jacobs, and J. N. Kok. Duality beyond sober spaces: Topological spaces and observation frames. *Theoret. Comput. Sci.*, 151, 1995.

[11] M. Bonsangue and A. Kurz. Duality for logics of transition systems, 2004. Draft, electronically available.

[12] A. Chagrov and M. Zakharyaschev. *Modal Logic*. Clarendon Press, 1997.

[13] C. Cîrstea. On expressivity and compositionality in logics for coalgebras. In *CMCS'03*, ENTCS, 2003.

[14] C. Cîrstea and D. Pattinson. Modular construction of modal logics. In *CONCUR'04*, LNCS 3170, 2004.

[15] G. Ferrari, U. Montanari, and M. Pistore. Minimizing transition systems for name passing calculi: A co-algebraic formulation. In *FoSSaCS'02*, LNCS 2303, 2002.

[16] J. Font, R. Jansana, and D. Pigozzi. A survey of abstract algebraic logic. *Studia Logica*, 74, 2003.

[17] N. Ghani, C. Lüth, F. de Marchi, and J. Power. Dualizing initial algebras. *Math. Structures Comput. Sci.*, 13, 2003.

[18] R. Goldblatt. Metamathematics of modal logic I. *Reports on Mathematical Logic*, 6, 1976.

[19] R. Goldblatt. Observational ultraproducts of polynomial coalgebras. *Ann. Pure Appl. Logic*, 123, 2003.

[20] B. Jacobs. Objects and classes, co-algebraically. In *Object-Orientation with Parallelism and Persistence*. Kluwer, 1996.

[21] B. Jacobs. Many-sorted coalgebraic modal logic: a model-theoretic study. *Theor. Inform. Appl.*, 35, 2001.

[22] P. Johnstone. *Stone Spaces*. Cambridge University Press, 1982.

[23] B. Jónsson and A. Tarski. Boolean algebras with operators, part 1. *Amer. J. Math.*, 73, 1951.

[24] M. Kick. Bialgebraic modelling of timed processes. In *ICALP'02*, LNCS 2380, 2002.

[25] J. Komenda and J. van Schuppen. Decentralized supervisory control with coalgebra. In *ECC'03*, 2003.

[26] M. Kracht. *Tools and Techniques in Modal Logic*. Elsevier, 1999.

[27] C. Kupke, A. Kurz, and D. Pattinson. Algebraic semantics for coalgebraic logics. In *CMCS'04*, ENTCS, 2004.

[28] C. Kupke, A. Kurz, and Y. Venema. Stone coalgebras. *Theoret. Comput. Sci.*, 327, 2004.

[29] A. Kurz. A co-variety-theorem for modal logic. In *Advances in Modal Logic 2*. CSLI, 2001.

[30] A. Kurz. Specifying coalgebras with modal logic. *Theoret. Comput. Sci.*, 260:119–138, 2001.

[31] A. Kurz and A. Palmigiano. Coalgebras and modal expansions of logics. In *CMCS'04*, ENTCS, 2004.

[32] L. Moss. Coalgebraic logic. *Annals of Pure and Applied Logic*, 96, 1999.

[33] L. Moss and I. Viglizzo. Harsanyi type spaces and final coalgebras constructed from satisfied theories. In *CMCS'04*, ENTCS, 2004.

[34] T. Mossakowski, H. Reichel, M. Roggenbach, and L. Schröder. Algebraic-coalgebraic specification in CoCASL. In *WADT'02*, LNCS 2755, 2003.

[35] A. Palmigiano. A coalgebraic semantics for positive modal logic. In *CMCS'03*, ENTCS, 2003.

[36] D. Pattinson. Coalgebraic modal logic: Soundness, completeness and decidability of local consequence. *Theoret. Comput. Sci.*, 309, 2003.

[37] D. Pattinson. Expressive logics for coalgebras via terminal sequence induction. *Notre Dame Journal of Formal Logic*, 45, 2004.

[38] H. Reichel. An approach to object semantics based on terminal co-algebras. *Math. Structures Comput. Sci.*, 5, 1995.

[39] M. Rößiger. From modal logic to terminal coalgebras. *Theoret. Comput. Sci.*, 260, 2001.

[40] J. Rothe, H. Tews, and B. Jacobs. The coalgebraic class specification language CCSL. *J. Univ. Comp. Sci.*, 7, 2001.

[41] J. Rutten. Automata and coinduction - an exercise in coalgebra. In *CONCUR'98*, LNCS 1466, 1998.

[42] J. Rutten. Universal coalgebra: A theory of systems. *Theoret. Comput. Sci.*, 249, 2000.

[43] J. Rutten. Coinductive counting with weighted automata. *J. Autom. Lang. Comb.*, 8, 2003.

[44] D. Turi and G. Plotkin. Towards a mathematical operational semantics. In *LICS'97*, 1997.

[45] J. van den Berg and B. Jacobs. The loop compiler for java and jml. In *TACAS'01*, LNCS 2031, 2001.

[46] Y. Venema. *Handbook of Modal Logic*, chapter Algebras and Coalgebras. To appear. Electronically available.

[47] S. J. Vickers. *Topology Via Logic*. Cambridge University Press, 1989.

[48] E. Vink and J. Rutten. Bisimulation for probabilistic transition systems: A coalgebraic approach. *Theoret. Comput. Sci.*, 221, 1999.