Exam paper for the PhD course on Interactions, automata, and names

Emilio Tuosto
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Department of Computer Science, University of Leicester, UK

1 Notation

- $\mathbb{N}$ is the set of natural numbers
- $\mathcal{N}$ is an countably infinite set of names
- $\mathcal{N}^*$ is the set of finite words on $\mathcal{N}$.

Question 1. Consider the following languages on $\mathcal{N}$:

$$\begin{aligned}
\mathcal{L}_{two} &\stackrel{\text{def}}{=} \left\{a_0 \cdots a_k \in \mathcal{N}^* \;\middle|\; k \in \mathbb{N} \wedge \forall 0 \le i \le k-1.\ a_i \neq a_{i+1}\right\} \\
\mathcal{L}_{all} &\stackrel{\text{def}}{=} \left\{a_0 \cdots a_k \in \mathcal{N}^* \;\middle|\; k \in \mathbb{N} \wedge \forall 0 \le i \neq j \le k.\ a_i \neq a_j\right\} \\
\mathcal{L}_{fst} &\stackrel{\text{def}}{=} \left\{a_0 \cdots a_k \in \mathcal{N}^* \;\middle|\; k \in \mathbb{N} \wedge \forall 1 \le i \le k.\ a_0 \neq a_i\right\} \\
\mathcal{L}_{lst} &\stackrel{\text{def}}{=} \left\{a_0 \cdots a_k \in \mathcal{N}^* \;\middle|\; k \in \mathbb{N} \wedge \forall 0 \le i \le k-1.\ a_i \neq a_k\right\} \\
\mathcal{L}^2_{all} &\stackrel{\text{def}}{=} \left\{w \circ v \;\middle|\; w, v \in \mathcal{L}_{all}\right\} \\
\mathcal{L}^2_{lst} &\stackrel{\text{def}}{=} \left\{w \circ v \;\middle|\; w, v \in \mathcal{L}_{lst}\right\} \\
\mathcal{L}_{tze} &\stackrel{\text{def}}{=} \bigcup_{i \in \mathbb{N}} \mathcal{L}_i \quad \text{where } \mathcal{L}_i \text{ is inductively defined by} \\
&\qquad \mathcal{L}'(\mathcal{H}) \stackrel{\text{def}}{=} \left\{a^i b^j c \in \mathcal{N}^* \;\middle|\; i, j \in \mathbb{N}, a \neq b, c \notin \mathcal{H} \cup \{a, b\}\right\}, \qquad \mathcal{H} \subseteq \mathcal{N} \\
&\qquad \mathcal{L}_0 \stackrel{\text{def}}{=} \mathcal{L}'(\emptyset) \\
&\qquad \mathcal{L}_{i+1} \stackrel{\text{def}}{=} \left\{w \circ v \;\middle|\; w \in \mathcal{L}_i, v \in \mathcal{L}'(|w|)\right\}, \qquad |w| \text{ is the set of names in } w
\end{aligned}$$

For each of the languages above

1. Tell if it is quasi-regular or not. For each quasi-regular language give an FMA that accepts it.
2. Tell if it is accepted by an FRA or not. For each language accepted by FRA, give an FRA that accepts it.

Question 2. Show that for every FRA with not global-freshness transitions there is an FMA which accepts the same language and viceversa.

Question 3. Consider UB-RE (unification-based regular expressions) and let $I(w)$ be the set of instances of the word w. Show that

$$I(w_1 \emptyset w_2) = I(w_1 w_2)$$
$$I(w_1 w_2) \neq I(w_1) I(w_2)$$

Question 4. FRA can be mapped on UHDA preserving languages. Give the details of the proof.

Question 5. Consider the following nominal regular expression with binders:

$$\langle_a \ \langle_b a\rangle_b^a \ (\langle_b b\rangle_b^a b)^* \ \rangle_a^a$$

compute its language and give an FPA that accepts such language.