Co-Algebraic Implementation of HD-automata

Partitioning Algorithm

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Italy

Plan of the talk

- Finite state verification...

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  - ...in open systems...

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  - ...in open systems...
  - ... via Semantic minimization
- Co-Algebraic Presentation of Transition Systems

Let's start...

Semantic Minimization

1990 Kannellakis, Smolka (IC 86)

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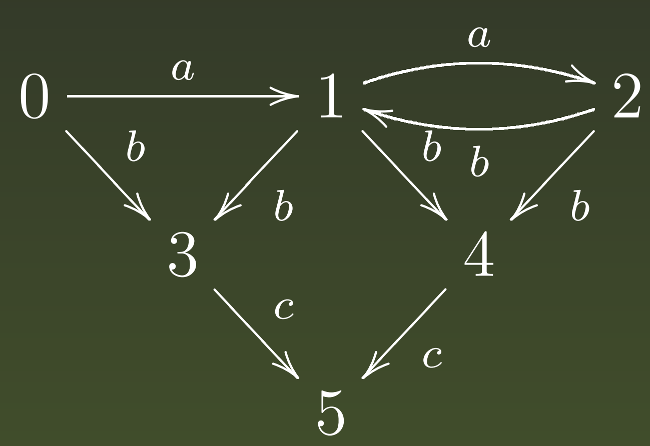

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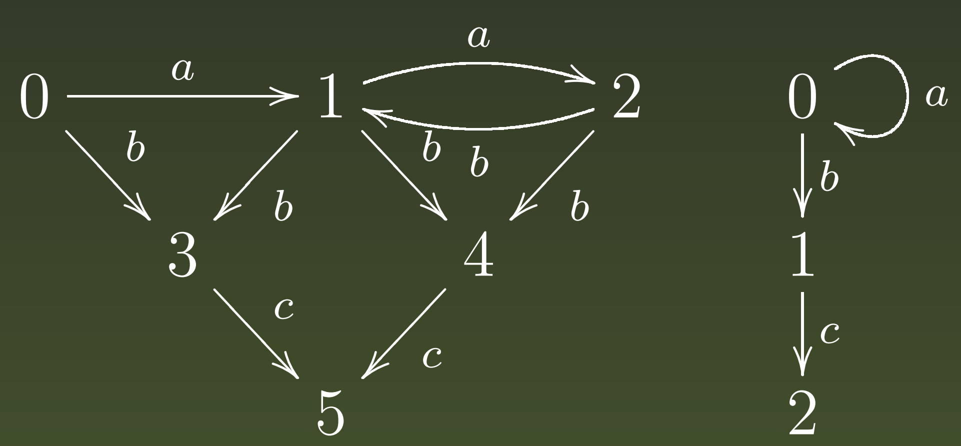

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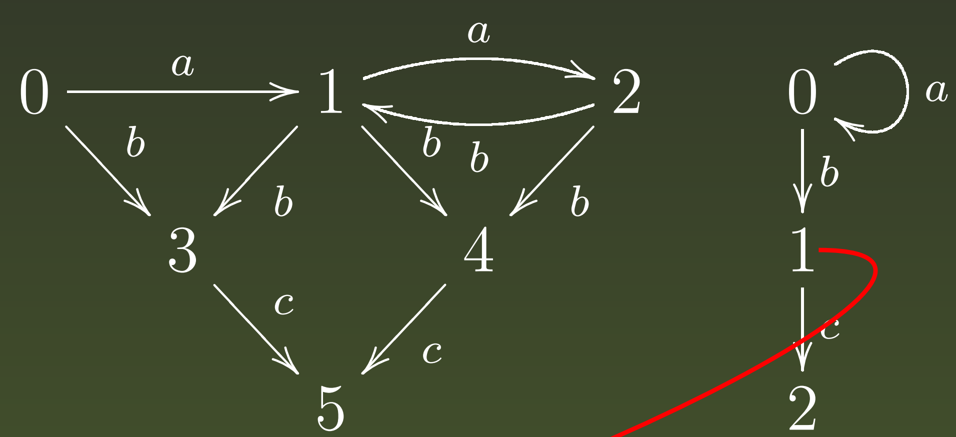

Mimimal automaton more suitable for verifi cation

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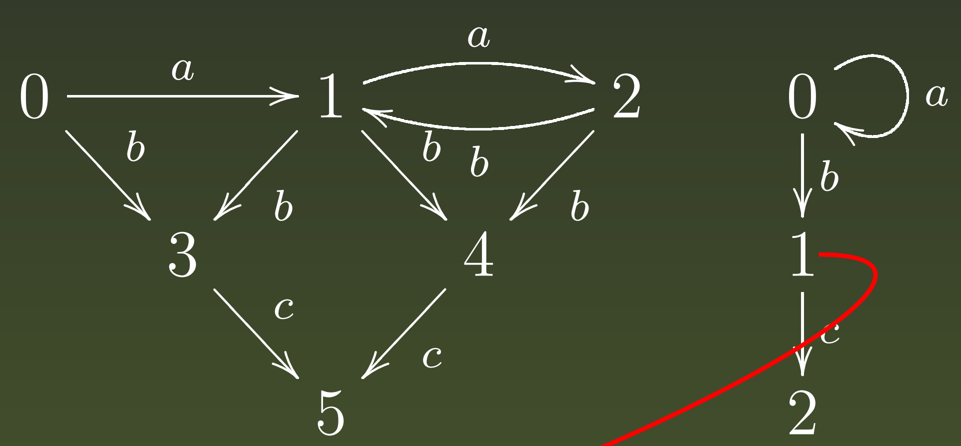

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Side effects:

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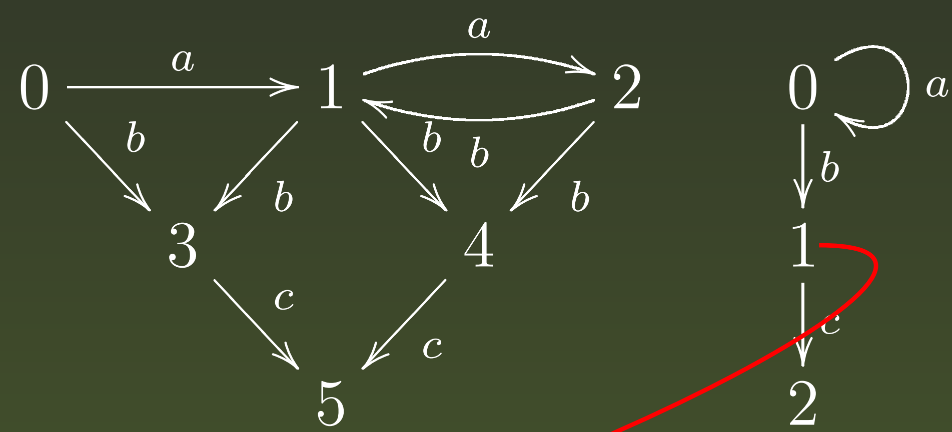

Mimimal automaton more suitable for verifi cation

Side effects:

- Checking bysimulation for free

$$\mathcal{A} \longrightarrow \mathcal{M}in \longleftarrow \mathcal{B} \qquad \Rightarrow \qquad \mathcal{A} \sim \mathcal{B}$$

- Composition of minimal components

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2. wireless communication
3. Dynamically reconfigurable systems
4. ...

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Open Systems via HD-Automata

- Open system modeled as HD-Automata

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- Extends classical automata

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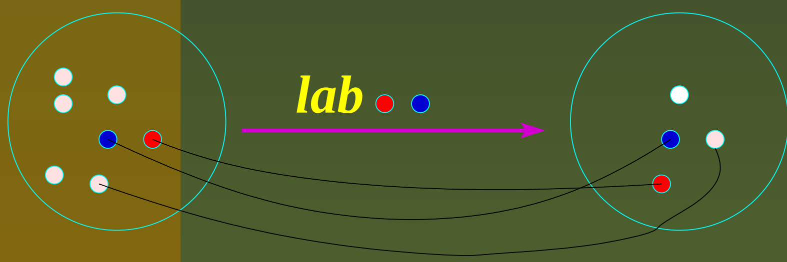
- Allocation/Deallocation of resources



- Names are local
- agents identified up-to name permutation
- name creation is well represented

Co-Algebraic Presentation of Automata

Some notations

Set collection of sets

$$q : Q \overset{not}{\equiv} q \in Q \quad (Q : \mathbf{Set})$$

Fun collection of arrows

$$H = \langle S : \mathbf{Set}, D : \mathbf{Set}, h : S \to D \rangle$$

$$S_H \overset{not}{=} S \quad D_H \overset{not}{=} D \quad h_H \overset{not}{=} h$$

$H; K$ denotes composition of $H, K : \mathbf{Fun}$ $(D_H = S_K)$

$$S_{H;K} = S_H$$

$$D_{H;K} = D_K$$

$$h_{H;K} = h_K \circ h_H$$

BasicDefinitions

Transition System

$T = (Q, L, \rightarrow)$

$\rightarrow \subseteq Q \times L \times Q$

Notation

$q \xrightarrow{l} q' \iff (q, l, q') \in \rightarrow$

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(step of a) bundle

$\beta = \langle d : \textbf{Set}, Step : \wp(L \times D)\rangle$

B collection of bundles

Lifting to functions of bundles...

$T(Q) = \{\beta : B | D_\beta = Q\}$

Let H : **Fun** we define $T(H)$: **Fun** s.t.

- $S_{T(H)} = T(S_H)$
- $D_{T(H)} = T(D_H)$
- $h_{T(H)} : \beta \mapsto \langle D_H, \{\langle l, h_H(q)\rangle | \langle l, q\rangle \in Step_\beta\rangle$

LTS are co-algebras

A LTS on states Q and labels L

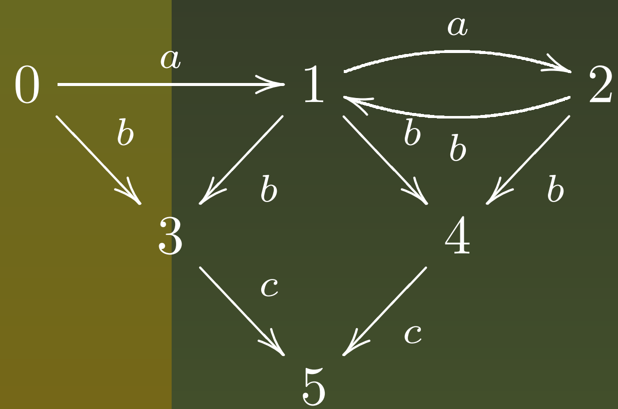
may be expressed as a

co-algebra K : **Fun**

such that

$$S_K = Q \quad D_K = T(Q)$$

Example



$$S_K = \{0, 1, 2, 3, 4, 5\}$$

$$K = \langle S_K, T(S_K), h_K \rangle$$

where

$$\begin{aligned}
h_K(0) &= \langle S_K, \{(a, 1), (b, 3)\} \rangle \\
h_K(1) &= \langle S_K, \{(a, 2), (b, 3), (b, 4)\} \rangle \\
h_K(2) &= \langle S_K, \{(a, 1), (b, 4)\} \rangle \\
h_K(3) &= \langle S_K, \{(c, 5)\} \rangle \\
h_K(4) &= \langle S_K, \{(c, 5)\} \rangle \\
h_K(5) &= \langle S_K, \emptyset \rangle
\end{aligned}$$