Concurrency, Causality and Reversibility: part 2

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April 2014

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Overview

Outline

Reversibility

Reversibility is very common in physics and biochemistry.
In nature reversibility underpins many mechanisms for achieving progress or change.

- e.g. building polymers, signal passing, catalysis

In artificial systems reversibility has a growing number of applications:

- saving energy
- debugging
- recovery from failure
  e.g. long-running transactions with compensations

Reachable states

We study reachable states.

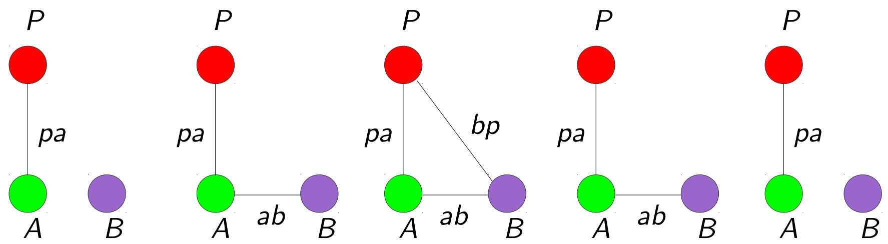
The most interesting are reachable states that are not forwards reachable.

Very common in mechanisms in nature that deliver change or progress while taking care of deadlock and failure.

Initially, we are more abstract and look at events, causation and conflict.
Then we consider modal logics for reversibility, and process calculi.

Motivation

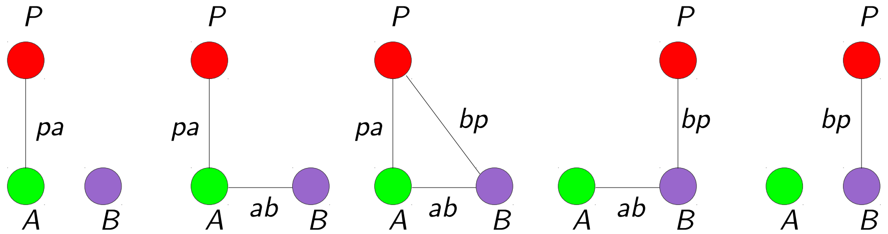
Biochemical bonds between molecules A, B and P:



Bond $P \cdot A$ causes $A \cdot B$ which causes $B \cdot P$
Bonds are dissolved in causal order: $P \cdot B$, $A \cdot B$ and then $A \cdot P$
Written abstractly as pa, ab, bp; $\underline{bp}$, $\underline{ab}$
Many applications of causal reversing in computer science.

Motivation

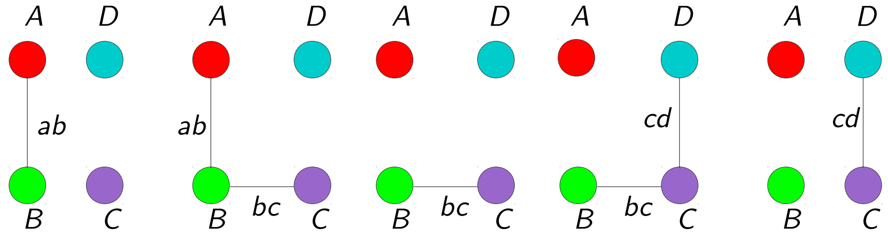
Signalling pathway: protein A passes signal P to protein B:



As before, bond $P \cdot A$ causes $A \cdot B$ which causes $B \cdot P$
Bonds are dissolved out of causal order: $P \cdot A$ then $A \cdot B$
Abstractly: pa, ab, bp; $\underline{pa}$, $\underline{ab}$
In nature out-of-causal-order reversing is very useful.
There are many forms of out-of-causal-order reversing, for example

Motivation

Catalyst chain: A helps B to bond with C, B helps C to bond with D:



ab causes bc; bc causes cd
Bonds are dissolved out of causal order: ab first, then bc
Complete computation: ab, bc, $\underline{ab}$, cd, $\underline{bc}$

Outline

Prime event structures

(Nielsen, Plotkin & Winskel)

Prime event structures (PES) is a triple $\mathcal{E} = (E, <, \sharp)$ where E is a set of events and

- $< \subseteq E \times E$ is the causality relation: an irreflexive partial order such that for every $e \in E$, $\{e' \in E : e' < e\}$ is finite;
- $\sharp \subseteq E \times E$ is the conflict relation: irreflexive, symmetric and *hereditary* with respect to $<$: if $a < b$ and $a \sharp c$ then $b \sharp c$ (all $a, b, c \in E$).

configurations X

- sets of events that have happened so far
- initially $\emptyset$
- conflict-free

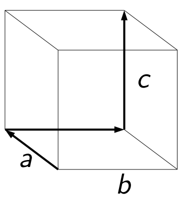
Examples

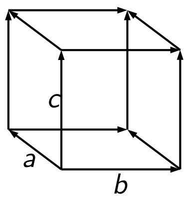
If $a < b$ and $b < c$ and there is no conflict, then $\emptyset \to \{a\} \to \{a, b\} \to \{a, b, c\}$. Depicted in the left cube by the sequence of thick arrows.

An alternative way to represent this computation is by abc.

Or, $\emptyset \xrightarrow{a} \{a\} \xrightarrow{b} \{a, b\} \xrightarrow{c} \{a, b, c\}$.

The cube on the right shows all possible executions when a, b and c are independent (here,causality and conflict are empty).



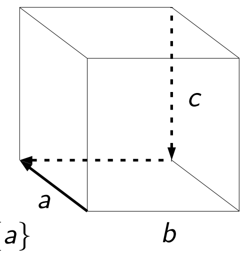


Examples

If we add $x \prec \underline{x}$, for all $x \in \{a, b, c\}$, and $\underline{a} \lhd b$, $\underline{b} \lhd c$ to $a < b < c$, then we achieve backtracking:

see the cube.

Note that only c can be undone in $\{a, b, c\}$ because $\underline{a} \lhd b$, $\underline{b} \lhd c$ and the presence of b, c in $\{a, b, c\}$ prevents undoing of a, b, respectively. Overall, we have



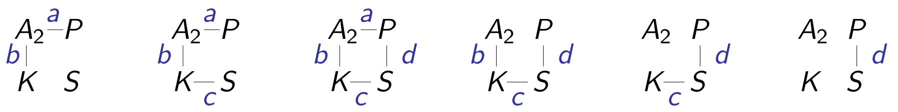
$\emptyset \to \{a\} \to \{a, b\} \to \{a, b, c\} \rightsquigarrow \{a, b, \} \rightsquigarrow \{a\}$

Basic catalytic cycle for protein substrate phosphorylation by a kinase.

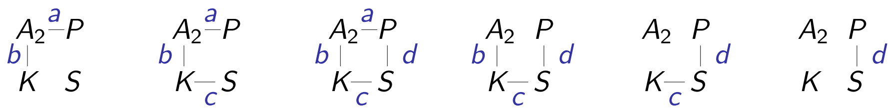
- Adenosine DiPhosphate (ADP) A_2
- Kinase K - the catalyst
- Substrate S
- Phosphate group P

Represent Adenosine TriPhospate (ATP) as $A_2 - P$.

P is transferred from A_2 to S.



Modelling



- Let events a, b, c, d represent (creation of) the bonds a, b, c, d.
- $a < b < c < d$

Reversing events

Simplest view

Reversing an event a means that a is removed from the current configuration

As if a had never occurred

- apart possibly from indirect effects, such as a having caused another event b before a was reversed.

Reverse causation

Undoing of a, b, c represented by $\underline{a}, \underline{b}, \underline{c}$

$$a, b, c, d, \underline{a}, \underline{b}, \underline{c}$$

Undone in same order as created - example of inverse causal order

Add to PES a new reverse causality relation $\prec$:

- $d \prec \underline{a}$, $d \prec \underline{b}$, $d \prec \underline{c}$ - need d to undo a, b, c
- also $a \prec \underline{a}$, $b \prec \underline{b}$ and $c \prec \underline{c}$

We do not include $d \prec \underline{d}$, since d is irreversible here.

Prevention

Extend PES further with a prevention relation $\triangleright$:

- $a \triangleright \underline{b}$ prevents undoing of b while a is present
- similarly $b \triangleright \underline{c}$

Get the desired ordering of $\underline{a}, \underline{b}, \underline{c}$.

Then $(\{a, b, c, d\}, \{a, b, c\}, <, \sharp, \prec, \triangleright)$ (with empty conflict $\sharp$) is a reversible PES (RPES).

Transitions

Forward transitions between configurations are

$$(\emptyset \rightarrow) \quad \{a\} \rightarrow \{a, b\} \rightarrow \{a, b, c\} \rightarrow \{a, b, c, d\}$$

and reverse transitions are

$$\{a, b, c, d\} \rightarrow \{b, c, d\} \rightarrow \{c, d\} \rightarrow \{d\}$$

Remark

There is a deficiency in the RPES solution in that, for example, a can occur again (so to speak) in configurations $\{b, c, d\}, \{c, d\}, \{d\}$.

We shall remedy this later by adding asymmetric conflict.

Conflict inheritance

Conflict inheritance - part of the definition of PES

- if $a < b$ and $a \sharp c$ then $b \sharp c$

Suppose a is reversible. If $a < b$ and $a \sharp c$:

$$\emptyset \rightarrow \{a\} \rightarrow \{a, b\}$$

Now undo a and c can be enabled:

$$\{a, b\} \rightarrow \{b\} \rightarrow \{b, c\}$$

So in reversible PES we do not require conflict inheritance with $<$.

Sustained causation

In PES, if $a < b$ then any configuration X which contains b will also contain a.
No longer holds in general in our reversible setting.

Sustained causation:

- $a \ll b$ means that $a < b$ and b prevents $\underline{a}$ (written $b \triangleright \underline{a}$).

So a cannot reverse until b reverses.

Configuration structures

Configuration structures

- introduced by van Glabbeek & Goltz (2001, part of work on refinement going back to 1989)
- later generalised by van Glabbeek & Plotkin

A configuration structure is a pair $\mathcal{C} = (E, \mathsf{C})$ where E is a set of events and $\mathsf{C} \subseteq \mathfrak{P}(E)$ is a set of configurations.

For $X, Y \in \mathsf{C}$, we let $X \to Y$ if $X \subseteq Y$ and for every Z, if $X \subseteq Z \subseteq Y$ then $Z \in \mathsf{C}$.
Idea: all the (possibly infinitely many) events in $Y \setminus X$ are independent, and so can happen as a single step.

Instead of $X \to Y$, we can write $X \xrightarrow{A} Y$ where $A = Y \setminus X$.

The reversible case

Note that if $Y = X \cup \{a\}$ and $X, Y \in \mathsf{C}$ then $X \to Y$.

This may no longer hold in the reversible setting.

As an example, let $E = \{a, b\}$ with $a < b$.
Then $\{b\}$ is not a possible configuration using forwards computation.
However if a is reversible:

$$\emptyset \xrightarrow{a} \{a\} \xrightarrow{b} \{a, b\} \xrightarrow{\underline{a}} \{b\}$$

Thus both $\emptyset$ and $\{b\}$ are configurations, but we do not have $\emptyset \xrightarrow{b} \{b\}$.

Configuration systems

A configuration system is a quadruple $\mathcal{C} = (E, F, \mathsf{C}, \to)$ where

- E is a set of events
- $F \subseteq E$ are the reversible events
- $\mathsf{C} \subseteq \mathfrak{P}(E)$ is the set of configurations
- $\to \subseteq \mathsf{C} \times \mathfrak{P}(E \cup \underline{F}) \times \mathsf{C}$ is a labelled transition relation such that if $X \xrightarrow{A \cup \underline{B}} Y$ then
  - $A \cap X = \emptyset$ and $B \subseteq X \cap F$ and $Y = (X \setminus B) \cup A$;
  - for every $A' \subseteq A$ and $B' \subseteq B$ we have $X \xrightarrow{A' \cup \underline{B'}} Z \xrightarrow{(A \setminus A') \cup (\underline{B \setminus B'})} Y$ (where $Z = (X \setminus B') \cup A' \in \mathsf{C}$).

Concurrent enabling: if $X \xrightarrow{A \cup \underline{B}} Y$ then all possible splits into sub-steps are enabled.

Mixed transitions

Transition $X \xrightarrow{A \cup \underline{B}} Y$ is mixed if both A and B are non-empty.

Example

$$\{a\} \xrightarrow{\{b, \underline{a}\}} \{b\}$$

This implies both

$$\{a\} \xrightarrow{b} \{a, b\} \xrightarrow{\underline{a}} \{b\} \text{ and } \{a\} \xrightarrow{\underline{a}} \emptyset \xrightarrow{b} \{b\}$$

Reachable configurations

Define various kinds of configuration (cf. van Glabbeek & Plotkin 2009):

Let $\mathcal{C} = (E, F, \mathrm{C}, \rightarrow)$ be a configuration system and let $X \in \mathrm{C}$.

- X is a forwards secured configuration if there is an infinite sequence of configurations $X_i \in \mathrm{C}$ $(i = 0, \ldots)$ with $X = \bigcup_{i=0}^{\infty} X_i$ and $X_0 = \emptyset$ and $X_i \xrightarrow{A_{i+1}} X_{i+1}$ with $A_{i+1} \subseteq E$;
- X is a reachable configuration if there is some sequence $\emptyset \xrightarrow{A_1 \cup \underline{B}_1} \cdots \xrightarrow{A_n \cup \underline{B}_n} X$ where $A_i \subseteq E$ and $B_i \subseteq F$ for each $i = 1, \ldots, n$;
- X is a forwards reachable configuration if there is some sequence $\emptyset \xrightarrow{A_1} \cdots \xrightarrow{A_n} X$ where $A_i \subseteq E$ for each $i = 1, \ldots, n$;
- X is a finitely reachable configuration if there is some sequence $\emptyset \xrightarrow{\alpha_1} \cdots \xrightarrow{\alpha_n} X$ where $\alpha_i \in E \cup \underline{F}$ for each $i = 1, \ldots, n$.

Reversible PES

A reversible prime event structure (RPES) is a sextuple $\mathcal{E} = (E, F, <, \sharp, \prec, \triangleright)$ where $(E, <, \sharp)$ is a pre-PES, $F \subseteq E$ are those events of E which are reversible, and

1. $\triangleright \subseteq E \times \underline{F}$ is the *prevention* relation;
2. $\prec \subseteq E \times \underline{F}$ is the *reverse causality* relation, where we require $a \prec \underline{a}$ for each $a \in F$, and also that $\{a : a \prec \underline{b}\}$ is finite and conflict-free for every $b \in F$;
3. if $a \prec \underline{b}$ then not $a \triangleright \underline{b}$;
4. $\sharp$ is hereditary with respect to *sustained causation* $\lll$: if $a \lll b$ and $a \sharp c$ then $b \sharp c$;
5. $\lll$ is transitive.

Each RPES $\mathcal{E}$ has an associated configuration system $C(\mathcal{E}) = (E, F, \mathrm{C}, \rightarrow)$.

Reachable configurations in RPESs

The causal depth of an event $e \in E$ in a pre-PES:

$$\mathrm{cdepth}(e) = \max\{\mathrm{cdepth}(e') + 1 : e' < e\}$$

where we conventionally let $\max(\emptyset) = 0$.

Causal depth is always finite, since each event has only finitely many causes.

Proposition

If $X \in \mathrm{C}$ is reachable then X is left-closed under $\lll$ and there is $k \in \mathbb{N}$ such that for all $e \in X$, $\mathrm{cdepth}(e) < k$.

Causal RPESs

Causal RPESs are ones where a reversible event can be reversed freely if all events it has caused have been reversed.

Definition

Let $\mathcal{E} = (E, F, <, \sharp, \prec, \triangleright)$ be an RPES.
We say that $\mathcal{E}$ is causal if for any $a \in E$, $b \in F$, we have both

1. $a \prec \underline{b}$ iff $a = b$ and
2. $a \triangleright \underline{b}$ iff $b < a$.

Any PES can be converted into a causal RPES, once we decide which events are to be reversible.

Cause-respecting RPES

We say that $\mathcal{E}$ is cause-respecting if for any $a, b \in E$, if $a < b$ then $a \ll b$, so that all causation is sustained causation.

- Weaker than causal

Theorem

Let $\mathcal{E}$ be a cause-respecting RPES and let $C(\mathcal{E}) = (E, F, \mathsf{C}, \rightarrow)$. If $X \in \mathsf{C}$ is reachable then X is forwards reachable (and left-closed).

Related to a result of Danos & Krivine for RCCS.

Outline

Asymmetric event structures

Asymmetric event structures $\mathcal{E} = (E, <, \lhd)$ (Baldan, Corradini & Montanari 2001) :
Like PESs, except that symmetric conflict $\sharp$ replaced by asymmetric conflict $\lhd$.

We write $a \lhd b$ iff $b \rhd a$.

Dual interpretation:

- $a \lhd b$ says that a weakly causes, or precedes event b, meaning that if both a and b occur then a occurred first
- $b \rhd a$ says that b prevents a, meaning that if b is present in a configuration then a cannot occur.

We have already used prevention $b \rhd \underline{a}$ on reverse events with RPESs.
$a \lhd b$ will give us greater control of forward events in the reversible setting.

Causation

In the reversible setting there is no good reason to insist on $<$ being transitive

- if $a < b < c$ then a may have been reversed after b occurs, and before c occurs.

Therefore, when defining RAESs we allow causation to be non-transitive.

Remarks

- This change is somewhat orthogonal to the move from symmetric to asymmetric conflict.
- Direct (or immediate) causation $\prec$ was used in flow event structures (Boudol & Castellani 1989) (with symmetric conflict $\sharp$).

Reversible AESs

We now generalise RPESs to the setting of asymmetric conflict $\lhd$ and not necessarily transitive causation $\prec$.
A reversible asymmetric event structure (RAES) is a quadruple $\mathcal{E} = (E, F, \prec, \lhd)$ where

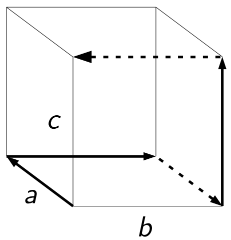
1. $\lhd \subseteq (E \cup \underline{F}) \times E$ is the precedence relation, which is irreflexive;
2. $\prec \subseteq E \times (E \cup \underline{F})$ is the direct causation relation, which is irreflexive and well-founded, and such that $\{e \in E : e \prec \alpha\}$ is finite and $\lhd$ is acyclic on $\{e \in E : e \prec \alpha\}$;
3. $a \prec \underline{a}$ for all $a \in F$;
4. if $a \prec \alpha$ then not $a \rhd \alpha$;
5. $a \prec\!\!\prec b$ implies $a \lhd b$, where sustained direct causation $a \prec\!\!\prec b$ means that $a \prec b$ and if $a \in F$ then $b \rhd \underline{a}$;
6. $\prec\!\!\prec$ is transitive;
7. if $a \sharp c$ and $a \prec\!\!\prec b$ then $b \sharp c$, where $\sharp$ is defined to be $\lhd \cap \rhd$.

Direct causation

- direct causation relation $\prec$
  combines forwards causation $<$ of (R)PESs and reverse causation $\prec$ of RPESs
- similarly precedence relation $\lhd$
  combines forwards precedence $\lhd$ of AESs and reverse prevention $\rhd$ of RPESs

Examples

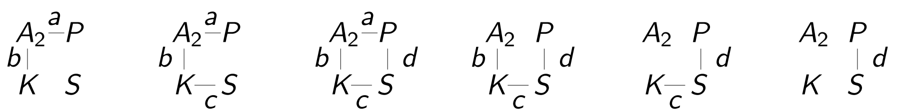
Out-of-causal-order reversing $a\ b\ \underline{a}\ c\ \underline{b}$.
We have $a \prec b \prec c$ but no $a \prec c$ ($\prec$ not transitive) and $a \prec \underline{a}$, $b \prec \underline{b}$ (no $c \prec \underline{c}$ since c irreversible). That a, b are undone only when b, c are present is ensured by $b \prec \underline{a}$, $c \prec \underline{b}$, respectively. To stop reversing b immediately after it occurs we add $\underline{b} \lhd a$. And, $a \lhd b$, $a \lhd c$ prevent a from re-occurring when b or c are present. Overall, we have



$$\emptyset \rightarrow \{a\} \rightarrow \{a,b\} \rightsquigarrow \{b\} \rightarrow \{b,c\} \rightsquigarrow \{c\}$$

Phosphorylation example revisited

We can now complete the modelling of our example.



$(\emptyset \xrightarrow{a}) \{a\} \xrightarrow{b} \{a,b\} \xrightarrow{c} \{a,b,c\} \xrightarrow{d} \{a,b,c,d\} \xrightarrow{\underline{a}} \{b,c,d\} \xrightarrow{\underline{b}} \{c,d\} \xrightarrow{\underline{c}} \{d\}$

Previous RPES:

- $a < b < c < d$ (transitive)
- $d \prec \underline{a}$, $d \prec \underline{b}$, $d \prec \underline{c}$ (need d to undo a, b, c)
- $a \prec \underline{a}$, $b \prec \underline{b}$ and $c \prec \underline{c}$
- $a \rhd \underline{b}$, $b \rhd \underline{c}$ (enforces order of $\underline{a}$, $\underline{b}$, $\underline{c}$)

With the RPES solution, for example, a can occur again in configurations $\{b,c,d\}, \{c,d\}, \{d\}$.

Modelling as RAES

Modify to get RAES:

- $a \prec b \prec c \prec d$ (no longer transitive)
- $a \lhd d$, $b \lhd d$, $c \lhd d$ (d prevents a, b, c from re-occurring)

  (In fact $a \lhd d$ is enough.)

Then $(\{a, b, c, d\}, \{a, b, c\}, \prec, \lhd)$ is the desired RAES.

So far

We have investigated reversibility in event structures with conflict and causation:

- Reversible form of prime event structures (RPES) where conflict inheritance no longer holds in general.
- More general model, reversible asymmetric event structures (RAES)
- Non-transitive causation
- Useful for controlled reversing, as distinct from processes computing freely either forwards or backwards
- reachable configurations

Next, we consider reversibility in event structures with enablings.

Outline

Event structures with enabling relation

Event structures were developed by Nielsen, Plotkin and Winskel in 1980s.
Events are things that happened. Typical events a, b, pa, ab.
Event structures (ES for short) are triples $\mathcal{E} = (E, \mathrm{Con}, \vdash)$ where

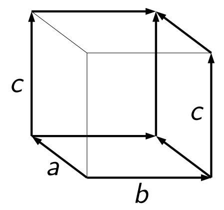
- E is a set of events
- $\mathrm{Con} \subseteq \mathcal{P}_{\mathrm{fin}}(E)$ is the consistency relation which is non-empty and satisfies downwards closure: $Y \subseteq X \in \mathrm{Con}$ implies $Y \in \mathrm{Con}$. It says which events can happen in the same computation
- $\vdash \subseteq \mathrm{Con} \times E$ is the enabling relation which satisfies the weakening condition: $X \vdash e$ and $X \subseteq Y \in \mathrm{Con}$ implies $Y \vdash e$ for all $e \in E$.

Example

Assume a, b, c have taken place. If $\{b, c\} \vdash d$ and $\{a, b, c, d\}$ is consistent then d can happen and afterwards we have a, b, c, d. And $\{a, b, c\} \rightarrow \{a, b, c, d\}$.

Disjunctive causation

a or b causes c. This is called disjunctive causation or inclusive or.
If we let the enabling relation as $\emptyset \vdash a$, $\emptyset \vdash b$, and $a \vdash c$ with $b \vdash c$, then we can deduce that $\{c\}$ is not a configuration since we have no $\emptyset \vdash c$. All other subsets of $\{a, b, c\}$ are configurations.



Configurations and computation

A configuration is a set of events that have happened:

X is a configuration if there is an infinite sequence $X_0, \ldots$ with $X = \bigcup_{n=0}^{\infty} X_n$, $X_0 = \emptyset$, $X_n \subseteq X_{n+1}$ and X_n consistent (all $n \in \mathbb{N}$), where for every $n \in \mathbb{N}$, and every $e \in X_{n+1} \setminus X_n$, there is a rule $X' \vdash e$ with $X' \subseteq_{\text{fin}} X_n$.

Computation: for configurations X, Y we have

$X \to Y$ if $Y \setminus X = \{e\}$ and $X' \vdash e$, for some e and $X' \subseteq_{\text{fin}} X$

Undoing of events

Common in natural and artificial systems, from polymer building to long-running transactions with compensation.
Causal-order reversing (which preserves causal order between events), and out-of-causal-order reversing.

Example

The ES for a.b is $\emptyset \vdash a$ and $a \vdash b$. We get $\emptyset \to \{a\} \to \{a, b\}$.
Causal-order reversing: $\{a, b\} \rightsquigarrow \{a\} \rightsquigarrow \emptyset$.
Out-of-causal-order reversing: $\{a, b\} \rightsquigarrow \{b\}$.

We shall define causal-order and out-of-causal-order reversing in the setting of event structures with enabling relation.

Reversible event structures

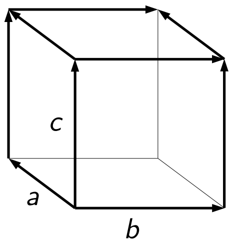
Reversible event structures (RES for short) are extensions of ESs with

- $\underline{E}$ the set of undone events, with typical elements $\underline{a}$, $\underline{b}$
- the new enabling relation:

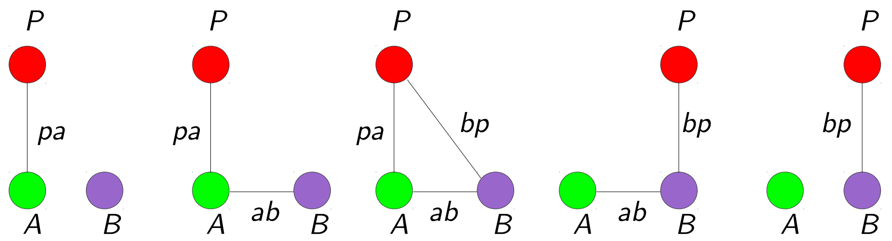
$$\begin{array}{ll} X \oslash Y \vdash a & \text{if } (X \cup \{a\}) \cap Y = \emptyset \\ X \oslash Y \vdash \underline{b} & \text{if } b \in X \end{array}$$

Resolvable conflict

There is a temporary conflict between events a and b which becomes resolved once a third event c occurs.
The enabling relation: $\emptyset \vdash c$, $\emptyset \oslash b \vdash a$ and $\emptyset \oslash b \vdash a$, meaning that initially, either a or b can take place if the other event is not present. We also have $c \vdash a$ and $c \vdash b$, which imply that a and b can happen after c.



Signalling pathway



Computation: pa, ab, bp; $\underline{pa}$, $\underline{ab}$

Example (Signalling pathway)

$$\{pa, ab, bp\} \vdash \underline{pa} \qquad \{ab, bp\} \oslash pa \vdash \underline{ab} \qquad bp \oslash \{pa, ab\} \vdash \underline{bp}$$

Computation: $\{pa, ab, bp\} \rightsquigarrow \{ab, bp\} \rightsquigarrow \{bp\} \rightsquigarrow \emptyset$.

Configurations

In the reversible setting configurations can grow as well as shrink.
Configurations can grow non-monotonically.

Example (Infinite non-monotonically growing sequence)

$$a_0,\ b_0,\ \underline{a_0},\ a_1,\ b_1,\ \underline{a_1},\ a_2, \ldots$$

The sets of events we get as computation progresses comprise an infinite sequence X_n where X_ns grow non-monotonically:

$$\begin{array}{l}
\emptyset, \\
\{a_0\},\ \{a_0, b_0\},\ \{b_0\}, \\
\{b_0, a_1\},\ ,\{b_0, a_1, b_1\}\ \{b_0, b_1\}, \\
\{b_0, b_1, a_2\},\ ,\{b_0, b_1, a_2, b_2\},\ \{b_0, b_1, b_2\}, \\
\ldots
\end{array}$$

Hence we need to work harder to define configurations.

Limits

Let $X_0, \ldots$ be an infinite sequence of subsets of E. We say that $X = \lim_{n\to\infty} X_n$ if for every $e \in E$:

1. e appears in finitely or cofinitely many X_n
2. $e \in X$ iff e appears in cofinitely many X_n

Note, $S \subseteq \mathbb{N}$ is cofinite if $\mathbb{N} \setminus S$ is finite.
For example, the sequence $\emptyset, \{a\}, \emptyset, \{a\}, \ldots$ has no limit.

Example (Infinite non-monotonically growing sequence)

Each a_i appears in finitely many sets X_n, while each b_j appears in cofinitely many X_ns.
Hence $\{b_i \mid i \in \mathbb{N}\}$ is the limit.

Configurations

X is a configuration if there is an infinite sequence $X_0, \ldots$ with $X = \lim_{n\to\infty} X_n$, $X_0 = \emptyset$ and $X_n \cup X_{n+1}$ consistent (all $n \in \mathbb{N}$), where for every $n \in \mathbb{N}$, and every $e^* \in X_{n+1} \setminus X_n$, there is a rule $X' \oslash Y' \vdash e^*$ such that:

1. $X' \subseteq_{\text{fin}} X_n$ and $X' + e^* \subseteq X_{n+1}$;
2. $Y' \cap (X_n \cup X_{n+1}) = \emptyset$.

e^* is either e or $\underline{e}$, and $\underline{e} \in X_{n+1} \setminus X_n$ means $e \in X_n \setminus X_{n+1}$.

Example (Infinite non-monotonically growing sequence)

Sets $\emptyset,\ \{a_0\},\ \{a_0, b_0\},\ \{b_0\},\ \{b_0, a_1\}, \ldots$ are configurations.
$\{b_i \mid i \in \mathbb{N}\}$ is a configuration.

Non-monotonic computation

Let $\mathcal{E} = (E, \mathrm{Con}, \vdash)$ where $E = \{a_i : i \in \mathbb{N}\} \cup \{b_j : j \in \mathbb{N}\}$ and $\mathrm{Con} = \{a_i, b_0, \ldots, b_j\}$ (any $i, j \in \mathbb{N}$) plus deducible subsets, with

$$\emptyset \vdash a_0 \quad a_i \vdash b_i \quad \{a_i, b_i\} \vdash \underline{a_i} \quad b_i \vdash a_{i+1} \quad (\text{all } i \in \mathbb{N})$$

The only possible computation sequence is

$$a_0,\ b_0,\ \underline{a_0},\ a_1,\ b_1,\ \underline{a_1},\ a_2,\ \ldots$$

with which we can associate a sequence $X_0 = \emptyset$, $X_1 = \{a_0\}, \ldots$. This has limit $\{b_j : j \in \mathbb{N}\}$, so is a configuration of $\mathcal{E}$.

Note that each a_i appears finitely often in the sequence X_n, while each b_j appears cofinitely often.

Results

1. RESs generalise ESs. Just take $\underline{E} = \emptyset$
2. Configurations of RESs generalise configurations of ESs
3. New $X \oslash Y \vdash a$ enablings are sufficiently powerful that we no longer require the consistency relation
4. Can encode PESs and RPESs, and well as AES and RAESs

References

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