An Analysis of Multi-objective Evolutionary Algorithms for Training Ensemble Models Based on Different Performance Measures in Software Effort Estimation

Leandro Minku, Xin Yao
{L.L.Minku,X.Yao}@cs.bham.ac.uk

CERCIA, School of Computer Science, The University of Birmingham
Introduction

- Software Effort Estimation (SEE).

- Machine learning for SEE.

- **Ensembles** of learning machines have been showing competitive performance in SEE.


- One of the keys for ensembles’ good performance: **diversity**, i.e., base models should make different errors on the same data points.

Multi-objective Ensembles for SEE

Different performance measures behave differently in SEE:

- **Mean Magnitude of the Relative Error (MMRE):**
  \[
  MMRE = \frac{1}{T} \sum_{i=1}^{T} MRE_i, \quad \text{where } MRE_i = \frac{|\hat{y}_i - y_i|}{y_i}; \quad \hat{y}_i \text{ is the predicted effort; and } y_i \text{ is the actual effort;}
  \]

- **Percentage of predictions within N\% of the actual value (PRED(N)):**
  \[
  PRED(25) = \frac{1}{T} \sum_{i=1}^{T} \begin{cases} 
  1, & \text{if } MRE_i \leq \frac{25}{100} \\
  0, & \text{otherwise}
  \end{cases}
  \]

- **Logarithmic Standard Deviation (LSD):**
  \[
  LSD = \sqrt{\frac{\sum_{i=1}^{T} (e_i + \frac{s^2}{2})^2}{T-1}}, \quad \text{where } s^2 \text{ is an estimator of the variance of the residual } e_i \text{ and } e_i = \ln y_i - \ln \hat{y}_i;
  \]
SEE as a Multi-objective Learning Problem

- Each performance measure is seen as an objective to be optimised (objective performance measure).
  

- A Multi-objective Evolutionary Algorithm (MOEA) can be used to create different SEE models based on these measures.

- The best model according to each objective performance measure can be used to compose an ensemble.
Multi-objective Evolutionary Algorithms (MOEAs)

Frequently guided by the concept of dominance: \( model_1 \) dominates \( model_2 \) if

- \( model_1 \) performs at least as well as \( model_2 \) in any objective performance measure; and
- \( model_1 \) performs better than \( model_2 \) in at least one objective performance measure.

In the end, MOEAs retrieve a set of nondominated solutions representing different trade-offs.
Previous work:

- SEE Models: Multi-Layer Perceptrons (MLPs).
- Objective performance measures: LSD, MMRE and PRED(25).

Why LSD, MMRE and PRED(25)?
Previous Work and Motivation

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- SEE Models: Multi-Layer Perceptrons (MLPs).
- Objective performance measures: LSD, MMRE and PRED(25).

Why LSD, MMRE and PRED(25)?

- Because they behave very differently.
- Diversity is important in ensembles.
- Optimising LSD, MMRE and PRED(25) resulted in improvements also in other measures such as Mean Absolute Error (MAE).
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- Optimising LSD, MMRE and PRED(25) resulted in improvements also in other measures such as Mean Absolute Error (MAE).

Using other performance measures would cause the MOEA to look for SEE models that specifically optimise these measures.
First Aim of the Work

Research Question 1

How *successful* would ensembles trained on these other objective performance measures be in comparison to LSD, MMRE and PRED(25)?

Research Question 2

Would training on other performance measures improve testing performance on these measures themselves? Or would using LSD, MMRE and PRED(25) still provide better testing performance on these other measures?

This will result in a better understanding of how to choose objective performance measures.
Data Sets

- Same as used in previous work.
- Cocomo81, nasa93, nasa, sdr, desharnais, 7 ISBSG data sets based on organization type.
- Attributes:
  - mostly cocomo attributes for PROMISE data (software reliability; size of database; complexity of product; software engineer capability; programming language; experience; etc);
  - functional size, development type and language type for ISBSG.
Besides LSD, MMRE and PRED(25), the following measures were considered, where $T$ is the number of projects, and $\hat{y}$ and $\bar{y}$ are the average predicted and average actual efforts:

1. $MAE = \frac{1}{T} \sum_{i=1}^{T} |\hat{y}_i - y_i|$;

2. $RMSE = \sqrt{\frac{\sum_{i=1}^{T} (\hat{y}_i - y_i)^2}{T}}$;

3. $Corr = \frac{\sum_{i=1}^{T} (\hat{y}_i - \bar{y})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{T} (\hat{y}_i - \bar{y})^2 \sum_{i=1}^{T} (y_i - \bar{y})^2}}$;

4. $MdAE = \text{Median} \{|\hat{y}_i - y_i| / 1 \leq i \leq T\}$;

5. $MdMRE = \text{Median} \{MRE_i / 1 \leq i \leq T\}$.

6. $StdDev = \sqrt{\frac{1}{T-1} \sum_{i=1}^{T} (\hat{y}_i - MAE)^2}$
Sets of Objective Performance Measures

Four sets of three measures were used:

- \{LSD, MMRE, PRED(25)\}: from previous work, known to behave diversely.
- \{MAE, RMSE, StdDev\}: intuitive in terms of evaluation.
- \{MAE, RMSE, Corr\}: includes correlation.
- \{MdAE, MdMRE, RMSE\}: contains median error measures.

Previous results suggest that HaD-MOEA copes well with three objectives.

Experimental Setup

- Test performance: median over 30 runs, using 9 performance measures.
- Holdout: 10 projects for testing, remaining for training, except for Sdr.
- Parameters:
  - Same as in the previous work.
  - These were shown to outperform several other approaches.
  - Default parameters for HaD-MOEAs.
  - Parameters more likely to obtain good results for MLP.
- Comparison of ensembles trained on different sets of measures: Friedman tests across data sets with level of significance of 0.05.
Results

We want to know (1) how successful ensembles created based on other performance measures are, and (2) whether using a certain measure as objective will result in best test performance on this measure.

Friedman detected statistically significant difference among ensembles for 6 out of 9 test performance measures: 

- LSD, MMRE, MAE, Corr, MdAE and MdMRE.
- \{LSD, MMRE, PRED(25)\} was ranked first for all these measures.

<table>
<thead>
<tr>
<th>MAE – p-value &lt; 0.0001*</th>
<th>Avg. Rank</th>
<th>Std. Dev. Rank</th>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.62</td>
<td>0.87</td>
<td>{LSD, MMRE, PRED}</td>
</tr>
<tr>
<td><strong>3.46</strong></td>
<td></td>
<td>0.78</td>
<td>{MAE, RMSE, Corr}</td>
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<tr>
<td>3.00</td>
<td>1.08</td>
<td>1.08</td>
<td>{MAE, RMSE, StdDev}</td>
</tr>
<tr>
<td>1.92</td>
<td>0.64</td>
<td></td>
<td>{MdAE, MdMRE, RMSE}</td>
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<table>
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<tr>
<th>MdMRE – p-value &lt; 0.0001*</th>
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<tbody>
<tr>
<td>Avg. Rank</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1.23</td>
</tr>
<tr>
<td>3.15</td>
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<tr>
<td>3.23</td>
</tr>
<tr>
<td>2.38</td>
</tr>
</tbody>
</table>
Results

- \{\text{MdAE, MdMRE, RMSE}\} usually obtained similar ranking to the best.

- However, still statistically significantly worse in terms of LSD and MdMRE.
  - Confirmed with Wilcoxon tests.

- \{\text{MAE, RMSE, Corr}\} was ranked last for all measures but MdMRE, where it was very similar to the last.
Results

Research Question 1

How successful would ensembles trained on these other objective performance measures be in comparison to LSD, MMRE and PRED(25)?

The other ensembles investigated here were not so successful as \{LSD, MMRE, PRED(25)\} in terms of several performance measures.
Results

Research Question 2

Would training on other performance measures improve testing performance on these measures themselves? Or would using LSD, MMRE and PRED(25) still provide better testing performance on these other measures?

Training on a particular objective measure did not necessarily lead to best performance on that measure itself. \{LSD, MMRE, PRED(25)\} obtained generally better results, even for the measures that it did not use as objectives.
Insight on Choosing Objective Performance Measures

If we understand why the results above were obtained, then we can get an insight into how objective performance measures should be chosen.

Probable reason:

- Ensembles different from \{LSD, MMRE, PRED(25)\} suffered more with overfitting.

- Diversity can help avoiding overfitting, and \{LSD, MMRE, PRED(25)\} may have generated more diversity.


S. Wang and X. Yao. Relationships Between Diversity of Classification Ensembles and Single-class Performance Measures. TKDE 2013
Checking Overfitting – Training performance

If training performance is very good, but testing performance is bad, that is a sign of overfitting.

Friedman detected statistically significant difference in 4 out of 9 measures on the training set:

- MAE, RMSE, Corr and StdDev.
- The approaches that use a certain objective performance measure achieved the best ranking, or ranking close to the best, on that measure.

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<tr>
<td></td>
<td>1.92</td>
<td>1.19</td>
<td>{MAE, RMSE, Corr}</td>
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<tr>
<td></td>
<td>1.69</td>
<td>0.63</td>
<td>{MAE, RMSE, StdDev}</td>
</tr>
<tr>
<td></td>
<td>2.77</td>
<td>0.44</td>
<td>{MdAE, MdMRE, RMSE}</td>
</tr>
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<table>
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<tr>
<td>Avg. Rank</td>
</tr>
<tr>
<td>3.77</td>
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<tr>
<td>1.31</td>
</tr>
<tr>
<td>2.08</td>
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<tr>
<td>2.85</td>
</tr>
</tbody>
</table>

Checking Overfitting – Training performance and diversity

- \{LSD, MMRE, PRED(25)\} was always ranked worse in terms of all training measures where there was statistically significant difference.
  - It obtained worst training, but best testing performance.
  - This indicates that it suffered less from overfitting.

Did \{LSD, MMRE, PRED(25)\} lead to more diversity?

- A MOEA generates diverse SEE models (several different trade-offs among measures).
- However, if its objectives are highly correlated, such trade-offs would not exist, as a solution that is good in one of them would also be good in the others.
- Check correlation between the performance measures to check the amount of diversity.
Checking Overfitting – Diversity

The maximum correlation between two training performance measures is higher for \{MAE, RMSE, Corr\} and \{MAE, RMSE, StdDev\}.

- These correspond exactly the the ensembles that obtained the worst testing performances, despite having obtained good training performance.

<table>
<thead>
<tr>
<th>{LSD, MMRE, PRED(25)}</th>
<th>{MAE, RMSE, Corr}</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSD vs MMRE</td>
<td>MAE vs RMSE</td>
</tr>
<tr>
<td></td>
<td>0.18</td>
</tr>
<tr>
<td>MMRE vs PRED(25)</td>
<td>RMSE vs Corr</td>
</tr>
<tr>
<td></td>
<td>0.33</td>
</tr>
<tr>
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<td>MAE vs Corr</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
</tr>
</tbody>
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<tr>
<td>MAE vs RMSE</td>
<td>MdAE vs MdMRE</td>
</tr>
<tr>
<td></td>
<td>0.97</td>
</tr>
<tr>
<td>RMSE vs StdDev</td>
<td>MdMRE vs RMSE</td>
</tr>
<tr>
<td></td>
<td>0.88</td>
</tr>
<tr>
<td>MAE vs StdDev</td>
<td>MdAE vs RMSE</td>
</tr>
<tr>
<td></td>
<td>0.74</td>
</tr>
</tbody>
</table>

When the correlation involved a measure to be maximised and a measure to be minimised, it was multiplied by -1.
Choosing Objective Performance Measures

Diversity among the objective performance measures should be a primary consideration in forming the group of objectives.
Second Aim of the Work

A MOEA can create several SEE models representing different trade-offs.

- Previous work used only the best according to the training performance on each objective measure.
- Including more models might provide better results.

Research Question 3

Is it better to use all SEE models produced by the MOEA, or solely the ones with the best training performance on each measure?

Result: there was no inherent advantage in using more SEE models in comparison to using the best model according to each measure.
Conclusions

- Study of MOEAs to generate ensembles of SEE models based on four combinations of objective performance measures.
  - \{LSD, MMRE, PRED(25)\} lead to a good level of diversity, and thus was more successful in terms of several performance measures.
- Using a certain objective measure does not necessarily lead to the best test performance on that measure.
  - Probable reason: overfitting.
- Diversity may help avoiding overfitting, and should be a primary consideration when choosing objective performance measures.
- No inherent advantage was found in using another scheme than the best model according to each measure to choose models to compose the ensemble.
The following should be further investigated:

- Other objective performance measures and combinations.
- Impact of using other MOEAs and parameters.
- Use of MOEAs for creating other types of SEE models.
- The relationship between low ensemble diversity and overfitting.