Typos for Categories for Types

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This document contains a list of some typos and other corrections for the first printing of "Categories for Types," many of which are very minor. I have also suggested a few changes which improve the text, rather than just giving the most succint and absolute corrections. The changes take either the form

Page/Section of change *Brief comment on the change*

Replace: Text 1

With: Text 2

or

Page/Section of change *Explicit description of the change*

In the first case, Text 2 should replace its counterpart Text 1. Note that the *actual* changes to be made often amount to a small part of Text 1. I am reprinting enough of the original text to make it easy to locate the changes—so some corrections are not as lengthy as they may appear at first sight!

List of Changes

(1) **Page 12, Remark 1.3.2** *To emphasise that in "Categories for Types" lattices have top and bottom elements—there is no error in the original remark*

Replace: Because $x \wedge y = x$ and $x \vee y = y$ just in case $x \leq y$ in a poset X, one only needs to check the existence of binary meets and joins of non-comparable pairs of elements of X to see that X is a lattice (as well as checking that X has top and bottom elements). Note also that a complete lattice has top and bottom elements (being the join and meet of the empty subset).

With: Note that both lattices and complete lattices have top and bottom elements (being the meet and join of the empty subset respectively). Also, having finite meets and joins is equivalent to having binary meets and joins and top and bottom elements. Finally, because $x \wedge y = x$ and $x \vee y = y$ just in case $x \leq y$ in a poset X, one only needs to check the existence of binary meets and joins of non-comparable pairs of elements of X to see that X is a lattice (as well as checking that X has top and bottom elements).

(2) **Page 13, Example 1.3.4(1)** *The examples $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ do not have top elements (and only $\mathbb{N}$ has a bottom element).*

Replace: $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are lattices with their usual order, where binary meets and joins are given by taking minima and maxima; for example in $\mathbb{N}$, $3 \wedge 5 = \min\{3, 5\} = 3$. If we adjoin a top and bottom to the set $\mathbb{R}$ to obtain $\mathbb{R}^* \stackrel{\text{def}}{=} (\mathbb{R} \cup \{\infty\} \cup \{-\infty\}, \leq)$ where for every $r \in \mathbb{R}$ we have $-\infty \leq r \leq \infty$, then $\mathbb{R}^*$ is a complete lattice due to the completeness axiom for R. Here, of course, ∞ and $-\infty$

are the top and bottom elements of $\mathbb{R}$ respectively. Similarly any closed interval $[r_1, r_2] \subseteq \mathbb{R}$ is a complete lattice when its order is inherited from $\mathbb{R}$.

With: If we adjoin a top and bottom to the set $\mathbb{Q}$ of rational numbers to obtain $\mathbb{Q}^* \stackrel{\text{def}}{=} (\mathbb{Q} \cup \{\infty\} \cup \{-\infty\}, \leq)$ where for every $q \in \mathbb{Q}$ we have $-\infty \leq q \leq \infty$, then $\mathbb{Q}^*$ is a lattice, but not a complete lattice. If we likewise add a top and bottom to the set $\mathbb{R}$ of reals to obtain $\mathbb{R}^* \stackrel{\text{def}}{=} (\mathbb{R} \cup \{\infty\} \cup \{-\infty\}, \leq)$ where for every $r \in \mathbb{R}$ we have $-\infty \leq r \leq \infty$, then $\mathbb{R}^*$ is a complete lattice due to the completeness axiom for $\mathbb{R}$. Similarly any closed interval $[r_1, r_2] \subseteq \mathbb{R}$ is a complete lattice when its order is inherited from $\mathbb{R}$.

(3) **Page 13, Example 1.3.4(3)** *The given example is not a lattice.*

Replace: The down sets $\{A{\downarrow} \mid A \subseteq X\}$ of a poset X form a complete lattice via intersection and union.

With: The down-closed subsets $\{A \subseteq X \mid x' \leq x, x \in A \text{ implies } x' \in A\}$ of a poset X form a complete lattice with meets and joins given by intersection and union.

(4) **Page 13, Example 1.3.4(4)** *Note that the example is indeed a* complete *lattice.*

(5) **Page 15, Proposition 1.3.9** *In the first sentence, change "poset" to "set".*

(6) **Page 25, Example 1.5.3(2)** *Change "anti-chain" to "discrete poset".*

(7) **Page 26, Proof of Proposition 1.5.5** *The equality does not follow from Exercise 1.5.4 because the joins in the proof are over different indexing sets.*

Replace: for all directed subsets D of X. This equality follows from Exercise 1.5.4

With: for all directed subsets D of X, which follows from the definition of join—check this.

(8) **Page 29, Proof of Theorem 1.5.12, (ii)** *Change "It is simple..." to "It is a simple..."*

(9) **Page 31, Proof** *In the 4th line of the proof, change "implying that $x \wedge y$ is in" to "implying that $x \vee y$ is in".*

(10) **Page 33, 1.6(1)** *In the second line of the exercise, change "finite" to "compact".*

(11) **Page 38** *In the maths display towards the bottom of the page, add a comma between (a, d) and (a, e) .*

(12) **Page 42, Example 2.2.2(7)** *At the very end of the example, add in "and $f(e) = e$."*

(13) **Page 53** *At the end of the last line, delete "a" before "giving".*

(14) **Page 73, Discussion 2.9.1** *Near the end of the page, change "$h: \mathbb{Q} \to \mathbb{R}$ is determined" to "$h: \mathbb{Q} \to R$ is determined". Here, R is any commutative ring.*

(15) **Page 112, Discussion 2.12.6** *The final "f" in a display should be "u".*

Replace:

$$(u, f) : (C, A) \longrightarrow (C', A') \quad \longmapsto \quad f : C \to C'.$$

With:

$$(u, f) : (C, A) \longrightarrow (C', A') \quad \longmapsto \quad u : C \to C'.$$

(16) $\boxed{\textbf{Page 115, Exercise 8(b)}}$ *The formal definition of U is wrong*

$\boxed{\textsf{Replace:}}$ Let U be the disjoint union of the sets DI as I runs over the objects of $\mathbb{I}$; formally $U \stackrel{\mathrm{def}}{=} ob\,\mathbb{I} \times \bigcup\{\mathbb{DI} \mid \mathbb{I} \in \mathbb{I}\}$.

$\boxed{\textsf{With:}}$ Let U be the disjoint union of the sets DI as I runs over the objects of $\mathbb{I}$; formally $U \stackrel{\mathrm{def}}{=} \bigcup\{\{I\} \times DI \mid I \in ob\,\mathbb{I}\}$ and a typical element of U is a pair (I, x) where $x \in DI$.

(17) $\boxed{\textbf{Page 119}}$ *"Mac Lanes's" should be "Mac Lane's".*

(18) $\boxed{\textbf{Page 130}}$ *In second paragraph, change "non-empty" to "empty".*

(19) $\boxed{\textbf{Page 137, Discussion 3.7.1}}$ *In the second paragraph, the function symbol should have sorting $f{:}\, \alpha_1, \ldots, \alpha_a \to \beta$.*

(20) $\boxed{\textbf{Pages 144,145}}$ *Certain α's should be β's, and there are some missing types within denotational brackets.*

$\boxed{\textsf{Replace:}}$ This will certainly be the case if given any morphism $(\Gamma \mid M){:}\, \vec{\alpha} \to [\beta]$ we have $h_\beta \circ [\![\Gamma \vdash M{:}\, \alpha]\!]_{\mathbf{M}} = [\![\Gamma \vdash M{:}\, \alpha]\!]_{\mathbf{N}} \circ (\times_1^n h_{\alpha_i})$. We show this by induction on the derivation of $Sg \;\rhd\; \Gamma \vdash M{:}\, \alpha$.

(*Case* $\Gamma \vdash M{:}\, \alpha$ *is* $\Gamma, x{:}\, \alpha, \Gamma' \vdash x{:}\, \alpha$): We have

$$\begin{array}{rcl} h_\alpha \circ \pi_{[\![\alpha]\!]_{\mathbf{M}}} & = & \pi_{[\![\alpha]\!]_{\mathbf{N}}} \circ \langle (\times_1^n h_{\alpha_i})\pi_{[\![\Gamma]\!]_{\mathbf{M}}}, h_\alpha \circ \pi_{[\![\alpha]\!]_{\mathbf{M}}}, (\times_1^m h_{\alpha'_j})\pi_{[\![\Gamma']\!]_{\mathbf{M}}} \rangle \\ & = & \pi_{[\![\alpha]\!]_{\mathbf{N}}} \circ (\times_1^n h_{\alpha_i}) \times h_\alpha \times (\times_1^m h_{\alpha'_j}). \end{array}$$

(*Case* $\Gamma \vdash M{:}\, \alpha$ *is* $\Gamma \vdash f(\vec{M}){:}\, \alpha$): Suppose that the context Γ is given by $[x_1{:}\, \alpha'_1, \ldots, x_m{:}\, \alpha'_m]$, and that we have proved terms $\Gamma \vdash M_i{:}\, \alpha_i$. Then we have

$$\begin{array}{rcl} \multicolumn{3}{l}{h_\beta \circ [\![\Gamma \vdash f(M_1, \ldots, M_n){:}\, \alpha]\!]_{\mathbf{M}}} \\ & = & h_\beta \circ [\![f]\!]_{\mathbf{M}} \circ \langle [\![\Gamma \vdash M_1{:}\, \alpha_1]\!]_{\mathbf{M}}, \ldots, [\![\Gamma \vdash M_n{:}\, \alpha_n]\!]_{\mathbf{M}} \rangle \\ & = & [\![f]\!]_{\mathbf{N}} \circ (\times_1^n h_{\alpha_i}) \circ \langle [\![\Gamma \vdash M_1{:}\, \alpha_1]\!]_{\mathbf{M}}, \ldots, [\![\Gamma \vdash M_n{:}\, \alpha_n]\!]_{\mathbf{M}} \rangle \\ \textit{by induction} & = & [\![f]\!]_{\mathbf{N}} \circ \langle [\![\Gamma \vdash M_1]\!]_{\mathbf{N}} \circ (\times_1^m h_{\alpha'_j}), \ldots, [\![\Gamma \vdash M_n]\!]_{\mathbf{N}} \circ (\times_1^m h_{\alpha'_j}) \rangle \\ & = & [\![f]\!]_{\mathbf{N}} \circ \langle [\![\Gamma \vdash M_1]\!]_{\mathbf{N}}, \ldots, [\![\Gamma \vdash M_n]\!]_{\mathbf{N}} \rangle \circ (\times_1^m h_{\alpha'_j}) \\ & = & [\![\Gamma \vdash f(M_1, \ldots, M_n)]\!]_{\mathbf{N}} \circ (\times_1^m h_{\alpha'_j}). \end{array}$$

$\boxed{\textsf{With:}}$ This will certainly be the case if given any morphism $(\Gamma \mid M){:}\, \vec{\alpha} \to [\beta]$ we have $h_\beta \circ [\![\Gamma \vdash M{:}\, \beta]\!]_{\mathbf{M}} = [\![\Gamma \vdash M{:}\, \beta]\!]_{\mathbf{N}} \circ (\times_1^n h_{\alpha_i})$. We show this by induction on the derivation of $Sg \;\rhd\; \Gamma \vdash M{:}\, \beta$.

(*Case* $\Gamma \vdash M{:}\, \beta$ *is* $\Gamma, x{:}\, \beta, \Gamma' \vdash x{:}\, \beta$): We have

$$\begin{array}{rcl} h_\beta \circ \pi_{[\![\beta]\!]_{\mathbf{M}}} & = & \pi_{[\![\beta]\!]_{\mathbf{N}}} \circ \langle (\times_1^n h_{\alpha_i})\pi_{[\![\Gamma]\!]_{\mathbf{M}}}, h_\beta \circ \pi_{[\![\beta]\!]_{\mathbf{M}}}, (\times_1^m h_{\alpha'_j})\pi_{[\![\Gamma']\!]_{\mathbf{M}}} \rangle \\ & = & \pi_{[\![\beta]\!]_{\mathbf{N}}} \circ (\times_1^n h_{\alpha_i}) \times h_\beta \times (\times_1^m h_{\alpha'_j}). \end{array}$$

(*Case* $\Gamma \vdash M{:}\, \beta$ *is* $\Gamma \vdash f(\vec{M}){:}\, \beta$): Suppose that the context Γ is given by $[x_1{:}\, \alpha'_1, \ldots, x_m{:}\, \alpha'_m]$, and that we have proved terms $\Gamma \vdash M_i \colon \alpha_i$. Then we have

$$\begin{array}{lcl}
h_\beta \circ [\![\Gamma \vdash f(M_1,\ldots,M_n)\colon \beta]\!]_{\mathbf{M}} & & \\
& = & h_\beta \circ [\![f]\!]_{\mathbf{M}} \circ \langle [\![\Gamma \vdash M_1\colon \alpha_1]\!]_{\mathbf{M}}, \ldots, [\![\Gamma \vdash M_n\colon \alpha_n]\!]_{\mathbf{M}} \rangle \\
& = & [\![f]\!]_{\mathbf{N}} \circ (\times_1^n h_{\alpha_i}) \circ \langle [\![\Gamma \vdash M_1\colon \alpha_1]\!]_{\mathbf{M}}, \ldots, [\![\Gamma \vdash M_n\colon \alpha_n]\!]_{\mathbf{M}} \rangle \\
\textit{by induction} & = & [\![f]\!]_{\mathbf{N}} \circ \langle [\![\Gamma \vdash M_1\colon \alpha_1]\!]_{\mathbf{N}} \circ (\times_1^m h_{\alpha'_j}), \ldots, [\![\Gamma \vdash M_n\colon \alpha_n]\!]_{\mathbf{N}} \circ (\times_1^m h_{\alpha'_j}) \rangle \\
& = & [\![f]\!]_{\mathbf{N}} \circ \langle [\![\Gamma \vdash M_1\colon \alpha_1]\!]_{\mathbf{N}}, \ldots, [\![\Gamma \vdash M_n\colon \alpha_n]\!]_{\mathbf{N}} \rangle \circ (\times_1^m h_{\alpha'_j}) \\
& = & [\![\Gamma \vdash f(M_1,\ldots,M_n)\colon \beta]\!]_{\mathbf{N}} \circ (\times_1^m h_{\alpha'_j}).
\end{array}$$

(21) **Page 148** *Towards the end of the proof:*

Replace: $[\![x\colon \Pi_1^n A_i \vdash id_{\Pi_1^n A_i}(\pi_1(x),\ldots,\pi_n(x))]\!]$

With: $[\![x\colon \Pi_1^n A_i \vdash id_{\Pi_1^n A_i}(\pi_1(x),\ldots,\pi_n(x))\colon \Pi_1^n A_i]\!]$

(22) **Page 149, Discussion 3.9.4** *The text indicates use of a generic model* **G***, not canonical model.*

Replace: ... in *Th*, because the soundness theorem gives

$$h = [\![x\colon A \vdash h(x)\colon B]\!]_{\mathbf{G}} = [\![x\colon A \vdash g(f(x))\colon B]\!]_{\mathbf{G}} = gf.$$

With: ... in *Th*, because (using the canonical model $\mathbf{M}$ of $Th(\mathcal{C})$ in $\mathcal{C}$) the soundness theorem gives

$$h = [\![x\colon A \vdash h(x)\colon B]\!]_{\mathbf{M}} = [\![x\colon A \vdash g(f(x))\colon B]\!]_{\mathbf{M}} = gf.$$

(23) **Page 165, Discussion 4.4.1** *Make changes at "three" places, add an op, change α to unit, change two $\Box$s to $\times$s:*

Replace:

$$\Phi_{A,B} : \mathcal{C}(-,A) \longrightarrow \mathcal{C}(-,B) : \mathcal{C} \longrightarrow \mathcal{S}et.$$

$m \stackrel{\text{def}}{=} [\![\Gamma \vdash M\colon \alpha]\!]$

$$(\Phi_{A,B})_C(\pi_A\langle m,n\rangle, \pi_B\langle m,n\rangle) = (\Phi_{A,B})_{A\Box B}(\pi_A,\pi_B) \circ \langle m,n\rangle,$$

that is $(\Phi_{A,B})_C(m,n) = (\Phi_{A,B})_{A\Box B}(\pi_A,\pi_B) \circ \langle m,n\rangle$.

With:

$$\Phi_{A,B} : \mathcal{C}(-,A) \longrightarrow \mathcal{C}(-,B) : \mathcal{C}^{op} \longrightarrow \mathcal{S}et.$$

$m \stackrel{\text{def}}{=} [\![\Gamma \vdash M\colon \mathit{unit}]\!]$

$$(\Phi_{A,B})_C(\pi_A\langle m,n\rangle, \pi_B\langle m,n\rangle) = (\Phi_{A,B})_{A\times B}(\pi_A,\pi_B) \circ \langle m,n\rangle,$$

that is $(\Phi_{A,B})_C(m,n) = (\Phi_{A,B})_{A\times B}(\pi_A,\pi_B) \circ \langle m,n\rangle$.

(24) $\boxed{\textbf{Page 166, Discussion 4.4.1}}$ *A $\Box$ should be $\times$.*

$\boxed{\text{Replace:}}$ $[\![\Gamma \vdash \langle M, N\rangle : A\Box B]\!] \stackrel{\text{def}}{=} \ldots$

$\boxed{\text{With:}}$ $[\![\Gamma \vdash \langle M, N\rangle : A \times B]\!] \stackrel{\text{def}}{=} \ldots$

(25) $\boxed{\textbf{Page 168, Discussion 4.4.1}}$ *Change part of the top of page: note that the equation (†) in the original text is removed.*

$\boxed{\text{Replace:}}$ Satisfaction of (5) requires that $(\Phi_{A,B})_C(ev_{A,B}(m \times id_A)) = m$ for every morphism $m: C \to A\Diamond B$ and from the naturality of $\Phi_{A,B}$ this holds just in case

$$(\Phi_{A,B})_{A\Diamond B}(ev_{A,B}) = id. \qquad (\dagger)$$

Now we come to the crucial point. If we define a natural transformation $\theta: \mathcal{C}(- \times A, B) \longrightarrow \mathcal{C}(-, A\Diamond B)$ by setting $\theta_C(f) \stackrel{\text{def}}{=} ev_{A,B} \circ (f \times id)$ the equations $(*)$ and $(\dagger)$ imply that θ is a natural bijection.

$\boxed{\text{With:}}$ Satisfaction of (5) requires that

$$(\Phi_{A,B})_C(ev_{A,B}(m \times id_A)) = m \qquad (\dagger)$$

for every morphism $m: C \to A\Diamond B$. Now we come to the crucial point. If we define a natural transformation $\Theta: \mathcal{C}(-, A\Diamond B) \longrightarrow \mathcal{C}(- \times A, B)$ by setting $\Theta_C(m) \stackrel{\text{def}}{=} ev_{A,B} \circ (m \times id)$ the equations $(*)$ and $(\dagger)$ imply that Θ is a natural bijection.

(26) $\boxed{\textbf{Page 173}}$ *The fourth line should say "It is easy...."*

(27) $\boxed{\textbf{Page 174, Exercise 4.7.2}}$ *This ought to refer to a cartesian closed category, not one with just finite products.*

$\boxed{\text{Replace:}}$ Try to think of a simple example of a $\lambda\times$-theory *Th*, a category with finite products $\mathcal{D}$ and a homomorphism $h: \mathbf{M} \to \mathbf{N}$ of models $\mathbf{M}$ and $\mathbf{N}$ of *Th* in $\mathcal{D}$.

$\boxed{\text{With:}}$ Try to think of a simple example of a $\lambda\times$-theory *Th*, a cartesian closed category $\mathcal{D}$ and a homomorphism $h: \mathbf{M} \to \mathbf{N}$ of models $\mathbf{M}$ and $\mathbf{N}$ of *Th* in $\mathcal{D}$.

(28) $\boxed{\textbf{Page 176, Proposition 4.8.2}}$ *In the statement of the proposition, change the two occurences of $\mathcal{C}$, to $\mathcal{D}$.*

(29) $\boxed{\textbf{Page179}}$ *In line 9 from the page bottom, delete a "we need".*

(30) $\boxed{\textbf{Page 185}}$ *I should preserve finite products.*

$\boxed{\text{Replace:}}$ Then a category $\mathcal{FC}$ is the *relatively free* cartesian closed category generated by $\mathcal{C}$ if there is a functor $I: \mathcal{C} \to \mathcal{FC}$ which satisfies the following two properties:

$\boxed{\text{With:}}$ Then a category $\mathcal{FC}$ is the *relatively free* cartesian closed category generated by $\mathcal{C}$ if there is a finite product preserving functor $I: \mathcal{C} \to \mathcal{FC}$ which satisfies the following two properties:

(31) $\boxed{\textbf{Page 187}}$ *Delete the spurious "is" on line 8.*

(32) **Page 190.** *The right hand downwards morphism of a diagram is wrong. Change the diagram to*

$$\begin{tikzcd}
(D \Rightarrow D') \times \Gamma(C \Rightarrow C') \arrow[r, "\pi_2"] \arrow[d, "\pi_1"'] & \Gamma(C \Rightarrow C') \arrow[d, "\lambda(\Gamma(ev)\circ \cong \circ(id_{\Gamma(C\Rightarrow C')} \times f))"] \\
D \Rightarrow D' \arrow[r, "id_D \Rightarrow f'"'] & D \Rightarrow \Gamma C'
\end{tikzcd}$$

(33) **Page 205** *In the middle of the page, change "raw term formed by replacing" to "raw type formed by replacing".*

(34) **Page 247** *At the end of the page,* Ω *is* $\{0, 1\}$ *where* $0 \leq 1$.

(35) **Page262, Discussion 5.7.1** *The first* $\mathcal{D}$ *to appear ought to read* $\mathcal{D}^{op}$.

(36) **Page 271,272** *Change a sentence at bottom of page 271, top of page 272.*

Replace: Let $(\Delta' \mid \Phi)$ be any object of $\mathcal{D}(\mathit{Type}^m, \mathit{Type})$, let f be as above, and define $[\![\Delta' \vdash \Theta]\!] \stackrel{\text{def}}{=} \theta$ and $[\![\Delta \vdash \Phi_i]\!] \stackrel{\text{def}}{=} \phi_i$.

With: Let $(\Delta' \mid \Theta)$ be any object of $\mathcal{D}(\mathit{Type}^m, \mathit{Type})$, let f be as above, and define $\theta \stackrel{\text{def}}{=} [\![\Delta' \vdash \Theta]\!]$ and $\phi_i \stackrel{\text{def}}{=} [\![\Delta \vdash \Phi_i]\!]$.

(37) **Page 291, Discussion 6.4.1(ii)** *I should be any set, not just a finite power of* U.

(38) **Page294** *In the bottom line,* M_α *should be* I_α.

(39) **Page 303** *In the middle of the page, the text should read "Take a filtered colimit....", not "Take a colimit...."*