Eccentricity Error Correction for Automated Estimation of Polyethylene Wear after Total Hip Arthroplasty

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Abstract. Acetabular wear of total hip replacements can be estimated from radiographs based on the apparent displacement of the femoral head relative to the acetabular cup. A wire marker is often attached to the polyethylene cup rim and its projection can be modelled as an ellipse. The centre of this ellipse is not the projection of the centre of the rim so its use as a reference point to measure wear can be problematic. The implications of the resulting eccentricity errors were investigated. The 3D poses of acetabular cups estimated from projected ellipse parameters were used to estimate error bounds and expected error values. The effect of correcting for these errors on wear measurements was investigated using standard clinical anteroposterior radiographs and an automated ellipse fitting method.

1 Introduction

Acetabular wear is the major cause of aseptic loosening of total hip replacements, leading to costly revision surgery and patient discomfort. Accurate estimation of wear is therefore important for early assessment of prosthesis performance. Wire markers are typically attached to the polyethylene acetabular component of the prosthesis so that both it and the metal femoral head component can be imaged effectively using standard x-ray imaging. In this paper, acetabular cups with rim wire markers are assumed. Displacement over time of the femoral head component relative to the acetabular cup indicates wear, the main component of which will be apparent in an anteroposterior (AP) radiograph.

The acetabular rim wire marker and the articular surface of the femoral head can be modelled as circular and spherical, respectively. The femoral head gives rise to an elliptical arc in the radiograph which is extremely close to circular. The acetabular rim marker, conversely, projects as a clearly non-circular ellipse due to (i) rotation in depth and (ii) translation away from the x-ray beam centre. Figure 1(a) shows a CAD model of a Zimmer ZCA acetabular cup undergoing rotation in depth.

The centre of the acetabular rim ellipse has been used as a reference point for the measurement of acetabular wear [1–3]. The implications of the fact that this point does not correspond to the projection in the image of the 3D centre of the rim wire marker (nor consistently to any point on the prosthetic structure) have not been properly explored. Figure 1(b) illustrates the *eccentricity error* that arises, defined as the distance between the centre of the projected ellipse and the projection of the centre of the circle. This paper explores the magnitude of this error and its effect on wear measurements made from standard clinical radiographs using an automated ellipse fitting method [1]. Sections 2 and 3 describe the methods used to estimate rotation in depth and eccentricity error. Section 4 presents an empirical investigation using standard clinical AP radiographs. Finally, some conclusions regarding clinical relevance and implications for related methods are drawn in Section 5.

2 Estimation of Rotation in Depth

The 3D pose of the acetabular rim circle can be estimated from its image provided that the film-focus distance, f, is known. Consider the cone of perspective projection that intersects the acetabular rim ellipse and the centre of projection. The acetabular pose can be found as the rotation such that the intersection of this cone with the image plane becomes a circle. Forsyth *et al.* [4] suggested computing this rotation in two stages. The first stage computes a rotation which centres the projected ellipse at the image centre with its major axis aligned with the x-axis. This is achieved by diagonalising the ellipse matrix. The second stage computes a rotation about the x-axis in order to obtain a circle. This method is now outlined more formally.

The image ellipse can be written in the form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \qquad (1)$$

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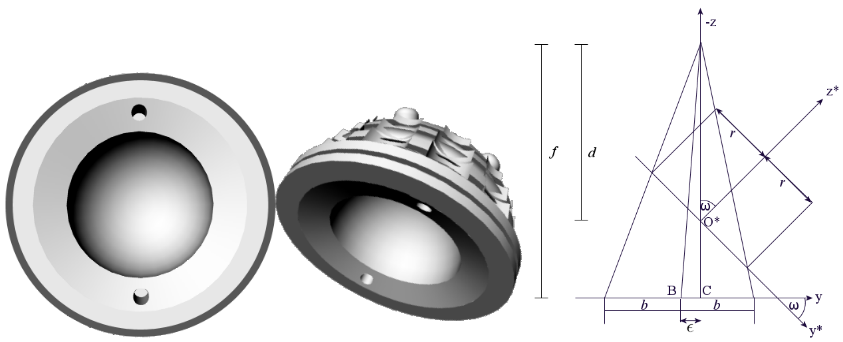


(a) (b)

Figure 1. (a) The acetabular cup with a circular rim and rotated in depth to produce an elliptical rim. (b) Side view showing the projection of the circle to the ellipse's minor axis. Eccentricity error, ϵ, is the distance between the projected ellipse centre B and the projection C of the rim centre.

If all distances are expressed as multiples of f, the equation of the cone (formed by this ellipse and the centre of projection) in matrix form is $\mathbf{x}^T\mathbf{E}\mathbf{x} = 0$, where $\mathbf{x} = [x, y, z]^T$ and $\mathbf{E}$ is the real, symmetric matrix:

$$\mathbf{E} = \begin{bmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{bmatrix} \tag{2}$$

The first rotation, $\mathbf{R}$, of this cone is obtained by matrix diagonalisation to obtain $\mathbf{E} = \mathbf{R}\mathbf{\Lambda}\mathbf{R}^T$, where $\mathbf{R}$ is the matrix whose columns are the eigenvectors and $\mathbf{\Lambda} = diag[\lambda_1, \lambda_2, \lambda_3]$ is the diagonal matrix of corresponding eigenvalues. Applying this rotation to the cone gives the new cone $\lambda_2 x^2 + \lambda_1 y^2 + \lambda_3 z^2 = 0$. Recall that a second rotation is then needed to obtain a circle in the image and that this rotation will be about the x-axis. Rotation by an angle ω about the x-axis would result in the cone:

$$\lambda_2 x^2 + (\lambda_1 \cos^2\omega + \lambda_3 \sin^2\omega)y^2 + (\lambda_1 \sin^2\omega + \lambda_3 \cos^2\omega)z^2 + 2(\lambda_1 + \lambda_3)\sin\omega\cos\omega yz = 0 \tag{3}$$

In order to obtain a circle, the x^2 and y^2 coefficients must be equated:

$$\lambda_2 = (\lambda_1 \cos^2\omega + \lambda_3(1 - \cos^2\omega)) \tag{4}$$

Rearranging gives the desired x-axis rotation:

$$\omega = \pm\cos^{-1}\sqrt{\frac{\lambda_2 - \lambda_3}{\lambda_1 - \lambda_3}} \tag{5}$$

In general there is a fourfold ambiguity in the value of ω corresponding to the four quadrants. It is assumed that ω lies in the first quadrant in the AP radiographs considered here.

3 Eccentricity Error

In standard clinical practice, the x-ray beam is centred on the symphysis pubis. Since the distance on the x-ray film between the symphysis pubis and the acetabulum is small relative to the film-focus distance, f, the beam centre is taken to be at the acetabulum in what follows. This simplifying assumption introduces an approximation error. Eccentricity of the rim marker ellipse is then explained in terms of a rotation in depth, ω, out of the plane of the radiograph.

The assumption that the acetabular rim marker centre is on the x-ray beam centre means that the eccentricity error's component in the direction of the ellipse's major axis is zero. An expression for the eccentricity error, ϵ, can therefore be computed as the midpoint of the two projected ellipse points that lie on the minor axis (see Figure 1(b)):

$$\epsilon = \frac{1}{2}\left(\frac{fr\cos\omega}{d + r\sin\omega} - \frac{fr\cos\omega}{d - r\sin\omega}\right) \tag{6}$$

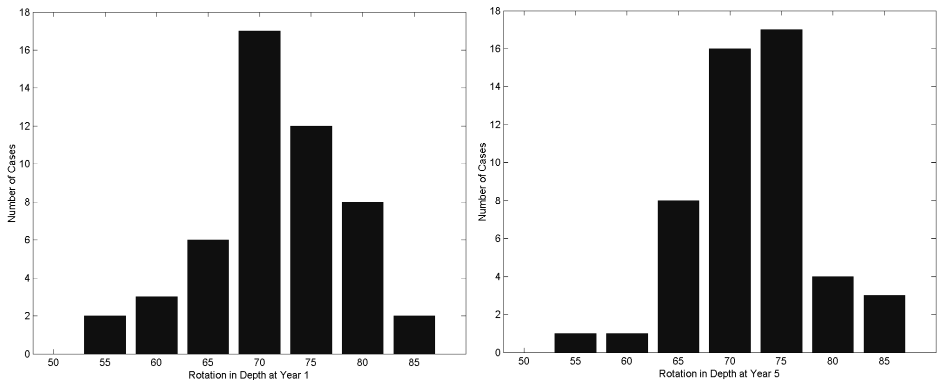
$$= \frac{-f\sin\omega\cos\omega}{\left(\frac{d}{r}\right)^2 - \sin^2\omega} \tag{7}$$

where r is the radius of the acetabular cup and d is the distance between the acetabular rim centre and the focus. In the experiments reported here, the rotation in depth, ω, was estimated using Equation (5).

4 Empirical Investigation

A test set of 50 cases consisting of year 1 and year 5 radiographs of cemented 28mm head diameter Zimmer CPT prostheses was obtained. These were standard clinical radiographs for which the exposure parameters f and d were not known. Each radiograph was digitised at 150 dpi. The femoral head and acetabular rim ellipses were localised automatically using the active ellipses method [1].

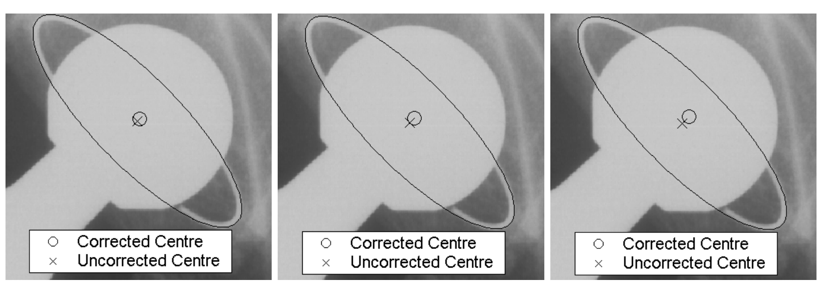
Rotations in depth were computed using Equation (5). Figure 2 shows the resulting distributions at year 1 and year 5 represented as histograms with a bin size of $5°$.



(a) (b)

Figure 2. Distribution of rotations in depth obtained for (a) Year 1 and (b) Year 5 radiographs.

Values for f and d are needed in order to compute the eccentricity error using Equation (7). Since these were unavailable, best, expected and worst case scenarios were investigated using values from the literature. In particular, Krismer *et al.* [2] report minimum, mean and maximum values for f of $900mm$, $1000mm$ and $1300mm$ respectively. They report object-*film* distances between $180mm$ and $270mm$. A middle value of $225mm$ was used here as an estimate for the expected value. Figure 3 shows the best, expected and worst case eccentricity errors for one of the radiographs. Displacing the ellipse centre along the minor axis by a distance ϵ recovers the projection of the rim centre and thus corrects for the eccentricity error.

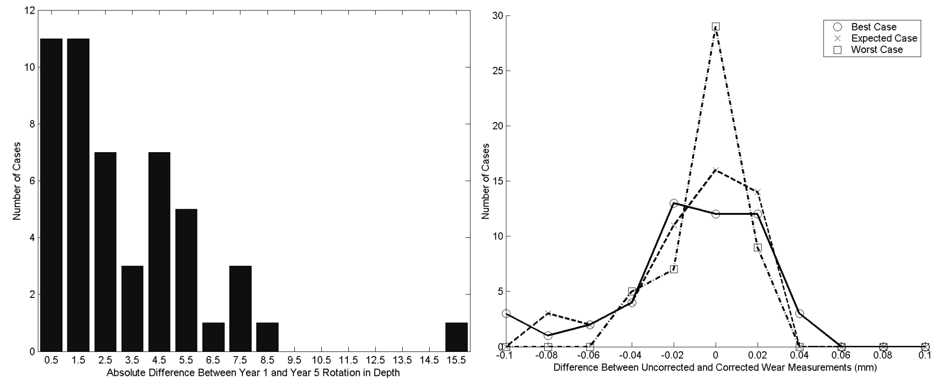


(a) (b) (c)

Figure 3. Visualisation of correction results where (a) $f = 1300mm$, $d = 1120mm$, (b) $f = 1000mm$, $d = 775mm$, and (c) $f = 900mm$, $d = 630mm$.

Acetabular wear can be estimated based on the change over time of the acetabular cup centre position relative to the femoral head centre. However, if the rim ellipse centre is used in place of the true centre, eccentricity error will affect wear estimates. If the eccentricity error is consistent over time, the wear error could be small. In order to investigate the possibility of such consistency, the absolute difference between rotations in depth at year 1 and year 5 was computed for each case. The resulting distribution is plotted in Figure 4(a).

The effect of correcting for eccentricity error upon wear estimates was computed using the worst, expected and best case exposure parameters. The distributions of the differences between wear estimates with and without correction are plotted in Figure 4(b). Standard deviations were found to be $0.016mm$, $0.026mm$ and $0.036mm$, with means of $-0.004mm$, $-0.008mm$ and $-0.012mm$ for the the best, expected and worst case exposure parameter values.



(a) (b)

Figure 4. (a) Distribution of absolute differences between Year 1 and Year 5 rotations in depth. (b) Binned distribution of differences between uncorrected and corrected wear values for different exposure parameters.

5 Discussion and Conclusions

The eccentricity error can have a significant effect on individual wear measurements producing changes as large as $0.05mm$, $0.08mm$ and $0.11mm$ using the best, expected and worst case exposure parameter values respectively. Linear wear rates have been found experimentally to range between $0.09mm/year$ and $0.25mm/year$ [5] so these errors can become clinically relevant. This serves to highlight the importance of recording f and d at acquisition in order that the errors can be corrected.

It is worth noting that as well as the duoradiographic approach to estimating wear (using two radiographs separated in time), there exist uniradiographic wear measurement techniques (that use only one radiograph). Such techniques include EBRA [2] and that of Eggli *et al.* [3]. While consistency of rotation in depth helps to mitigate the effect of eccentricity errors in the duo-radiographic approach, the effect of such errors on uniradiographic techniques is likely to be more pronounced.

Eccentricity error due to translation of the acetabular cup away from the beam centre was not considered in this study. Such translation introduces further error including, in general, a component in the direction of the ellipse's major axis. In standard clinical radiographs the beam centre often lies on the symphysis pubis which can be as much as 900 pixels away from the acetabular rim centre on a 150 dpi radiograph. If the location of the beam centre, or that of the symphysis pubis as an approximation to the beam centre, were known, the full eccentricity error could be estimated. Ahn *et al.* [6] provide a detailed treatment of the relevant geometry (using a camera model).

Acknowledgments

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