Eccentric Elliptical Contours in Total Hip Replacements

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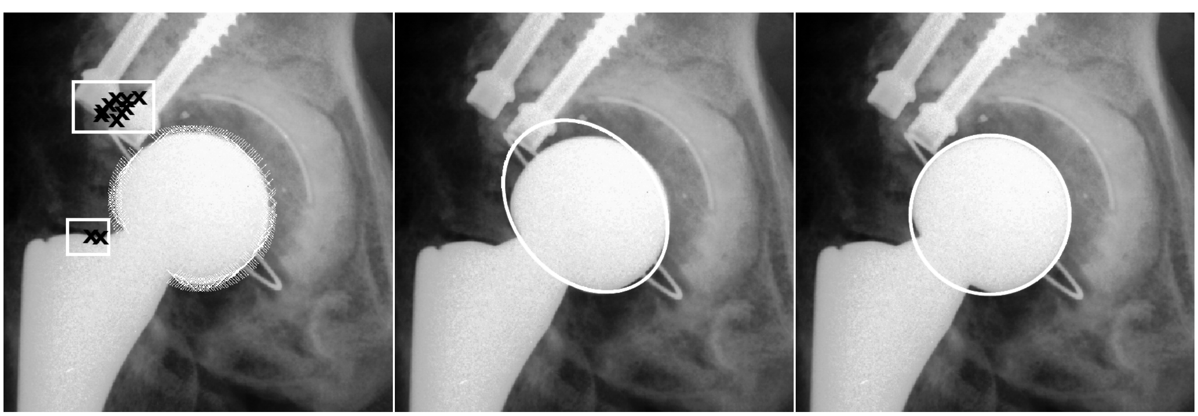
Abstract. The active ellipses method for assessing wear in total hip replacements uses robust ellipse fitting to estimate ellipses around the contours of the femoral head and acetabular rim wire marker. In the case of the latter these ellipses can be very eccentric and the standard algebraic distance was shown to be inadequate. Unlike in robust estimation of a line the geometric distance from an ellipse to a point is not trivial to compute and thus numerous error of fit functions have been created. In this work several of these error of fit functions are compared, including a geometric error of fit function, on both synthetic data and by using the active ellipses on a set of test radiographs containing eccentric rims.

The robust estimation using geometric error function was found to be the best performing in all but Gaussian noise (where a least squares fit using geometric error is the best) its performance was similar to that of a computationally cheaper error function known as the foci bisector distance to the extent where the two were almost interchangeable.

1 Introduction

The active ellipses method [1] for assessing wear in total hip replacements (THRs) uses robust ellipse fitting. Radiopaque debris, such as those seen in Figure 1a cause structured outlying points from which a standard least squares (LS) ellipse fit [2] generates erroneous results, as seen in Figure 1b. Most conventional ellipse fitting uses an algebraic error function causing problems with fitting eccentric ellipses such as those of the acetabular rim. LS with a geometric error function [3] shows a similar performance. Robust Least Median of Squares is desirable in this instance as it has a breakdown point of 50% outlying points. It can use different error of fit functions but the most obvious choice is the geometric distance of a data point to the closest point on the ellipse curve. Unlike in robust estimation of a line this is not trivial to compute and thus numerous computationally cheaper error functions [4] [5] have been considered in the past. Today's computational power and the availability of a geometric error algorithm have increased the feasibility of using the geometric distance. The performance of the geometric error was compared with that of other error functions. The reliability of active ellipse measurements using geometric, foci bisector distance and algebraic error functions was additionally examined.



(a) (b) (c)

Figure 1. (a) Data points found with a femoral head search with the Active Ellipses method, returning outliers (black points highlighted by white rectangle) (b) An incorrect LS fit to the data points (c) A robust LMedS fit capable of finding the solution in the presence of outliers

2 Materials and Methods

In order to assess the performance of the ellipse fitting algorithms synthetic data was created with known ellipse parameters and known eccentricity. Eccentricity is defined in this paper as $\sqrt{1-\frac{b^2}{a^2}}$ where a is the major subaxis

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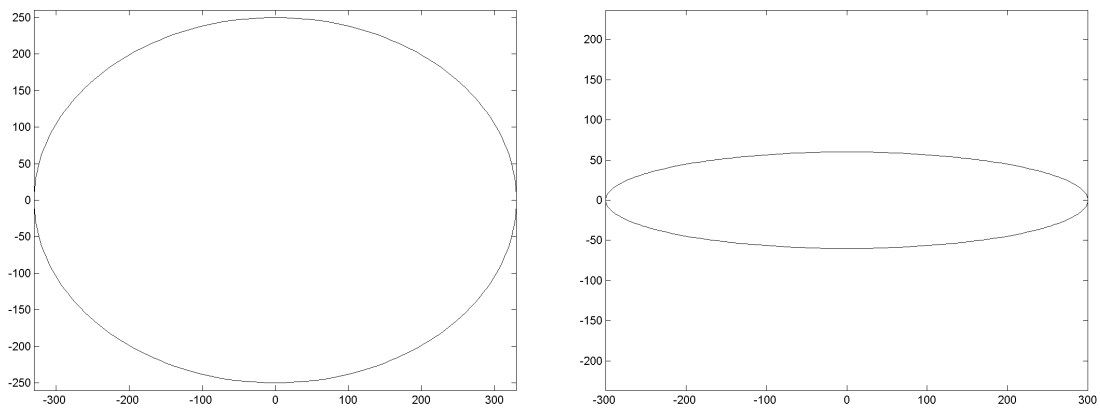
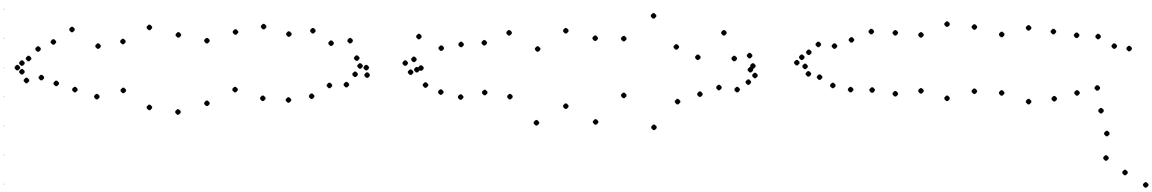


(a) (b)

Figure 2. (a) The less eccentric ellipse, $a = 330$, $b = 250$, centred at the origin without orientation (b) The eccentric ellipse, $a = 300$, $b = 60$, centred at the origin without orientation.



(a) (b) (c)

Figure 3. Data points created from the more eccentric ellipse parameters (a) corrupted by Gaussian Noise (σ=10) (b) 50% corrupted by Gaussian noise (σ=5) and remaining 50 % corrupted by Gaussian noise (σ=20) creating outlying points (c) corrupted by Gaussian noise (σ=5) with 20% noisy line arc added.

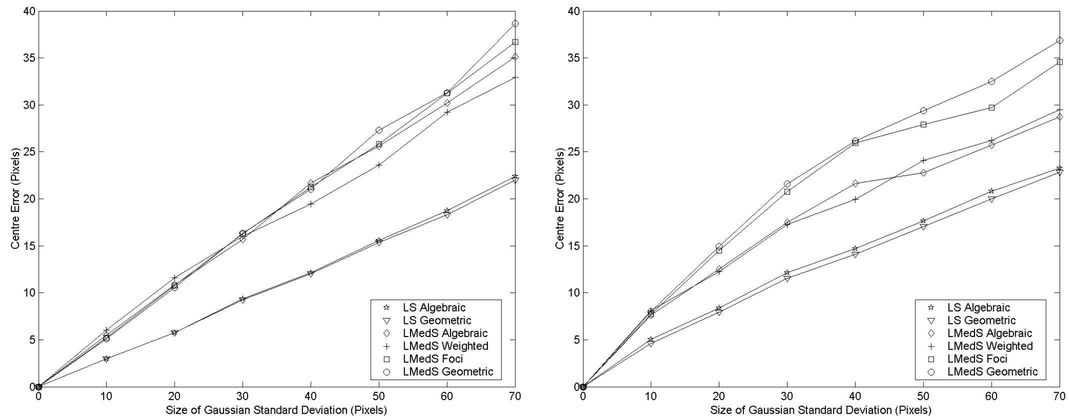
and b the minor subaxis of an ellipse. This data consisted of 38 points sampled along an elliptical arc from known parameters. 3 types of synthetic data were created, the first corrupted by Gaussian noise of varying standard deviation, the second contained a fixed number of points additionally corrupted by Gaussian noise with a large standard deviation and the third containing a percentage of structured outliers in the form of a noisy line. An example of each of these cases (with smaller standard deviation for visualisation) is shown in Figure 3. 500 sets of points were created for each level of probability of outliers. Two ellipses were considered with eccentricities of 0.67 and 0.98 (see Figure 2). LS fits using geometric and algebraic error functions and LMedS fits (with LS fine tuning) using algebraic, weighted algebraic by gradient, foci bisector distance and geometric error functions were considered. The euclidean distance between the original and recovered centre points were used as a score of performance.

The performance of the active ellipses method using the most promising error of fit functions was investigated using robust fitting on 19 radiographs containing Zimmer CPT prostheses with rim eccentricities of 0.96 and higher. Searches used the error functions to estimate five point fits twice in each image, but with no fine tuning to the initial estimates. Typical datasets have eccentricities of 0.8 to 1.

3 Results

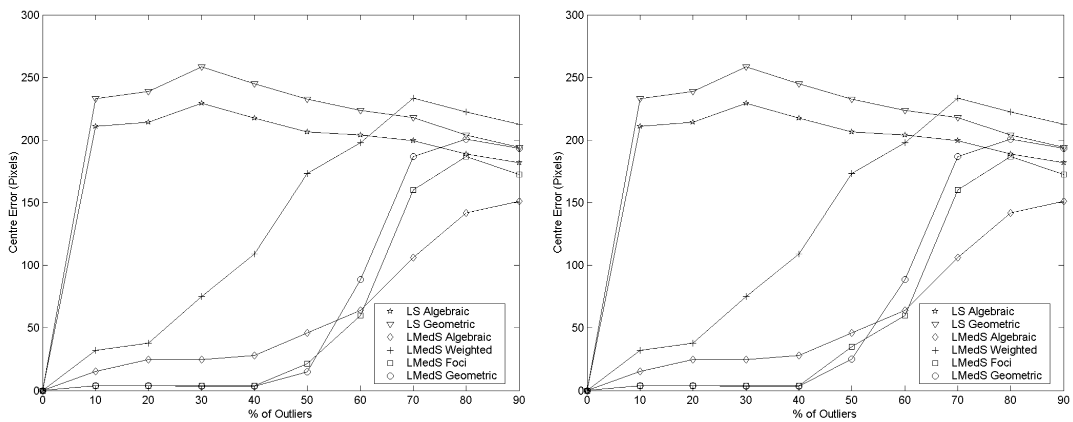
Figures 4-6 show the trimmed means of the centre errors for each of the synthetic data sets. In Figure 6 the centre error goes beyond the scale of the graph as selecting points on the noisy line resulted in extremely eccentric and erroneous ellipses being generated.

On the radiograph dataset LMedS Geometric failed 15 times out of 38, LMedS Foci failed 16 times and LMedS Algebraic failed 32 times. Given the difficulty of the dataset and the absence of an LS fine tune to inliers identified from the minimal subset (which increases performance of all three error functions) the LMedS Geometric result is encouraging. An example of the output of each of these algorithms is shown in Figure 7.



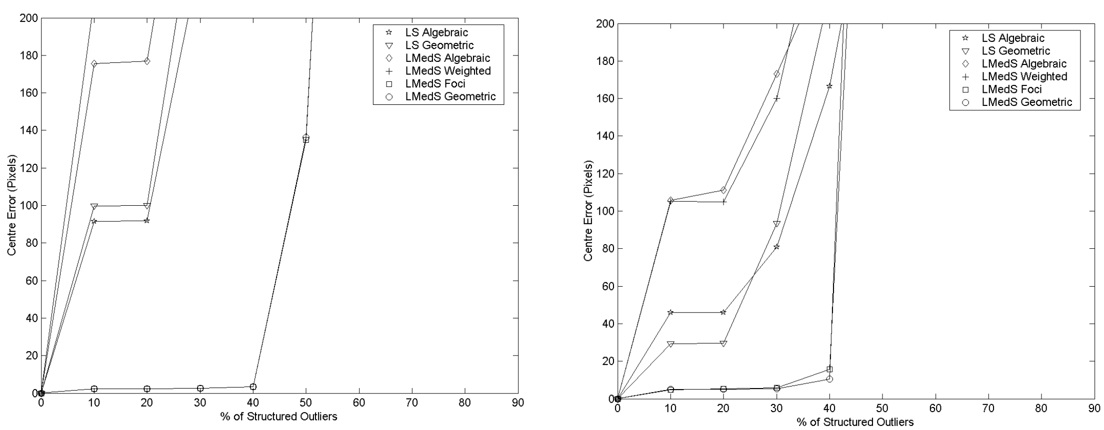
(a) (b)

Figure 4. Alpha trimmed mean (α = 0.1) (a) centre error of less eccentric ellipse synthetic data (b) centre error of more eccentric ellipse synthetic data when Gaussian noise is applied with varying σ



(a) (b)

Figure 5. Alpha trimmed mean (α = 0.1) (a) centre error of less eccentric ellipse synthetic data (b) centre error of more eccentric ellipse synthetic data when mixed Gaussian noise is added

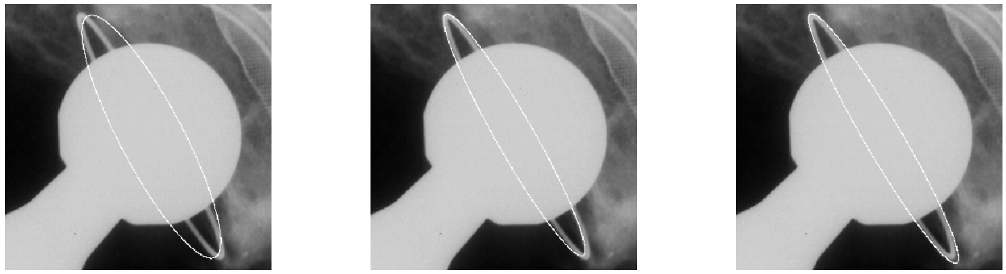


(a) (b)

Figure 6. Alpha trimmed mean (α = 0.1) (a) centre error of less eccentric ellipse synthetic data (b) centre error of more eccentric ellipse synthetic data when structured outlying noise is added

4 Discussion

In the presence of pure Gaussian noise, as shown in Figure 4, both LS algorithms outperform any of the robust LMedS algorithms. Least squares fitting assumes all noise is Gaussian and thus it is optimal under these circum-



(a) (b) (c)

Figure 7. Sample eccentric rim with (a) LMedS Algebraic fit (failure) (b) LMedS Foci Fit (success) (c) LMedS Geometric Fit (success)

stances. On the more eccentric ellipse the difference between the LS Algebraic and LS Geometric becomes more pronounced, with the LS Geometric performing better. None of the robust fittings perform well. LMedS Geometric and LMedS Foci perform the worst.

The plots in Figures 5 and 6 show the LMedS Foci and LMedS Geometric perform best on both datasets, with the only signficant difference in performance being the breakdown between 40% and 50% in Figure 6b. Data obtained beyond 50% is beyond the theoretical break-down points of the LMedS algorithm and can be assumed to be nonsense.

From the synthetic data it seems the LMedS Geometric performs slightly better on both noisy data. In Figure 6 the LMedS Algebraic and LMedS Weighted fits break down surprisingly early whilst both LS fits perform better. The LS methods fit to both the line and the elliptical arc, while LMedS Algebraic and Weighted seem to favour points on the noisy line. LS performs better on the more eccentric noisy line data as the fit to all the points is closer to the centre.

LMedS Geometric and Foci Bisector perform the best on the noisy datasets and thus were ran on the radiographic dataset. LMedS Algebraic was also used as it had been used as standard in previous works. LMedS Geometric made the most successful estimates, just outperforming LMedS Foci. LMedS Algebraic performed poorly, as can be seen in the sample outputs on Figure 7, where both LMedS Geometric and LMedS Foci succeeded.

These experiments demonstrate that the LMedS Geometric fit performs slightly better than the foci bisector distance approximation. However it is a computationally expensive method and in algorithms where speed is required the foci bisector distance is recommended as an error of fit function. The application of the geometric distance as an error function for LMedS ellipse fitting is novel.

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