

# Mining State-Based Models from Proof Corpora

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# Motivation

## Interactive Theorem Proving

The process of interacting with a computer to complete proofs.

User completes proof by entering a sequence of *tactics*.

Same task for novice and experienced users.

## How to proceed?

When manual intuition hasn't led to a proof:

- Automated tactics - `auto`, `firstorder`, `tauto`...
- Outsource to ATPs (`why3` in Coq, `sledgehammer` in Isabelle).
- Utilise existing proofs.
  - `search`, `searchAbout`, `searchPattern`.
  - ML4PG (by Heras and Komendantskaya)

# Existing Proofs

Examples where user has entered a correct sequence of tactics.



## The Coq Standard Library

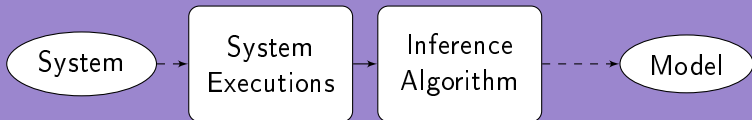
Here is a short description of the Coq standard library, which is distributed with the system. It provides a set of modules directly available through the `Require Import` command.

The standard library is composed of the following subdirectories:

- Init:** The core library (automatically loaded when starting Coq)  
 Notations Datatypes Logic Logic\_Type Peano Specif Tactics Wf (Prelude)
- Logic:** Classical logic and dependent equality  
 SetIsType Classical\_Pred\_Set Classical\_Pred\_Type Classical\_Prop Classical\_Type (Classical) ClassicalFacts  
 Decidable Eqdep\_dec EqdepFacts Eqdep JMeq ChoiceFacts RelationalChoice ClassicalChoice ClassicalDescription  
 ClassicalEpsilon ClassicalUniqueChoice Berardi Diaconescu Hurkens ProofIrrelevance ProofIrrelevanceFacts  
 ConstructiveEpsilon Description Epsilon IndefiniteDescription FunctionalExtensionality ExtensionalityFacts
- Structures:** Algebraic structures (types with equality, with order, ...). `DecidableType*` and `OrderedType*` are there only for compatibility.  
 Equalities EqualitiesFacts Orders OrdersTac OrdersAlt OrdersEx OrdersFacts OrdersLists GenericMinMax  
 DecidableType DecidableTypeEx OrderedType OrderedTypeAlt OrderedTypeEx
- Bool:** Booleans (basic functions and results)  
 Bool BoolEq DecBool IfProp Sumbool Zerob Bvector
- Arith:** Basic Peano arithmetic  
 Arith\_base Le Lt Plus Minus Mult Gt Between Peano\_dec Compare\_dec (Arith) Min Max Compare Div2 EqNat Euclid  
 Even Bool\_nat Factorial Wf\_nat
- PArith:** Binary positive integers  
 BinPosDef BinPos Pnat POrderedType (PArith)
- NArith:** Binary natural numbers  
 BinNatDef BinNat Nnat Ndigits Ndist Ndec Ndiv\_def Ngcd\_def Nsqrt\_def (NArith)

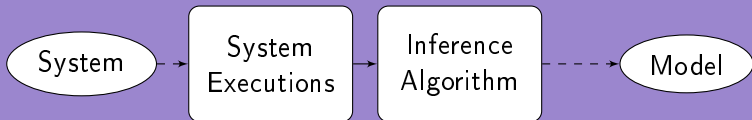
# Model Inference

## Software Engineering

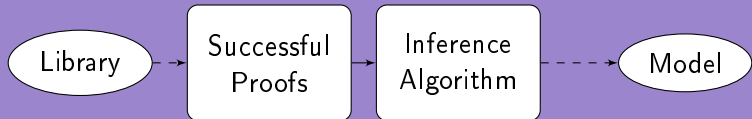


# Model Inference

## Software Engineering

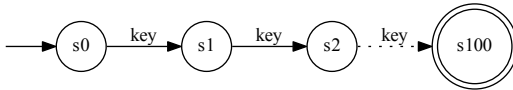


## Theorem Proving



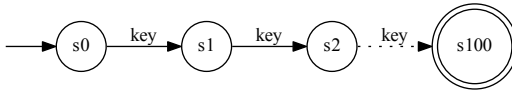
# FSMs and EFSMs Example

## Finite State Machine

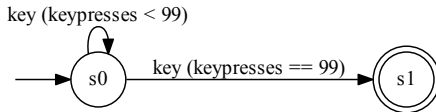


## FSMs and EFSMs Example

### Finite State Machine



### Extended Finite State Machine





# Modelling Proofs with State Machines - 1

Given the following examples:

induction n. simpl. trivial.

induction a. intros. trivial.

induction l. trivial.

induction m. trivial.

induction n. trivial.

induction l. simpl. trivial.

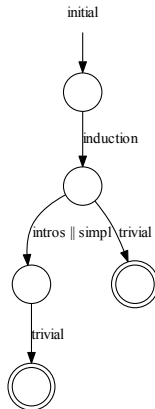
# Modelling Proofs with State Machines - 1

Remove the parameters:

```

induction. simpl. trivial.
induction. intros. trivial.
induction. trivial.
induction. trivial.
induction. trivial.
induction. simpl. trivial.
    
```

Inferred FSM

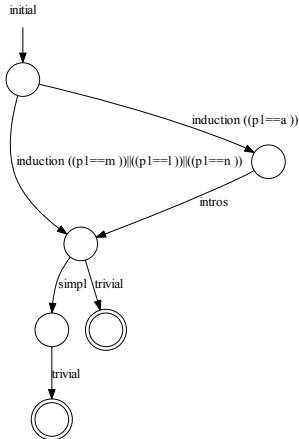


## Modelling Proofs with State Machines - 2

Given the following examples:

induction n. simpl. trivial.  
 induction a. intros. trivial.  
 induction l. trivial.  
 induction m. trivial.  
 induction n. trivial.  
 induction l. simpl. trivial.

Inferred EFSM



## Evaluation Process - Accuracy

### Sensitivity

Proportion of times a model correctly accept a valid sequence of tactics.

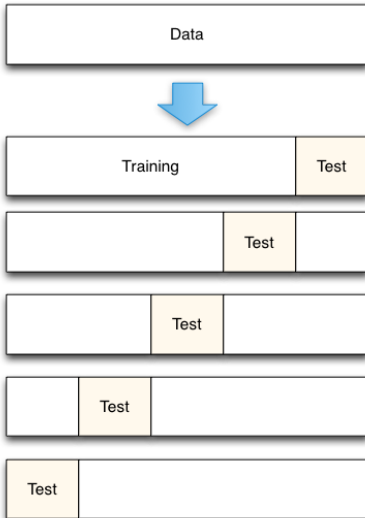
### Specificity

Proportion of times a model correctly rejects an invalid sequences of tactics.

Negative examples generated by:

- Randomizing valid tactic sequences
- Using proofs from different theories to the dataset

# Evaluation Process - k-folds cross validation



# Results

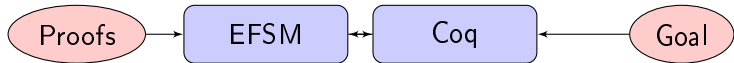
Data Set	Proofs	Sensitivity	Specificity
ListNat	70	0.84	0.81
Bool	100	0.95	0.55
Coqlib	100	0.22	0.96
Values	85	0.24	0.98

## Qualitative Value of Models

Can an inferred model be useful in proof development?

- Provides a visual interpretation of proofs
- Manually inspect the model.
- Automated application of EFSMs

# Automated Application



Proof attempt is made by search through inferred EFSM.

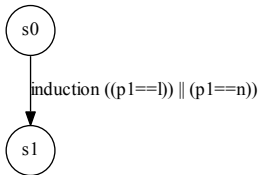


## Preliminary Results from Automated Application

Data Set	EFSM Success
ListNat	67%
Bool	30%
ConstructiveGeometry	35%
RegExp	25%
Float	48%

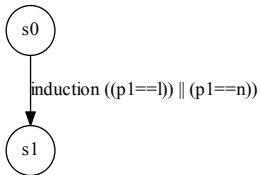
# Abstraction of labels

Without Types

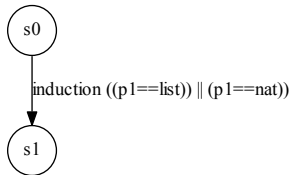


# Abstraction of labels

Without Types



With Types



## Incorporate Negative Informations

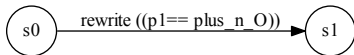
Currently, models are inferred from successful examples.

During a proof attempt, there may be a lot of negative examples - failed derivations.

Can we include this information in the model?

## Filtering of Existing Proofs

Combine our tool with clustering tool ML4PG (Heras and Komendantskaya).



If this fails, try lemmas in the same cluster as `plus_n_0`

# Conclusions

We have shown that:

- Model Inference can be applied to theorem proving
- Inferred models can be useful in proof development
- Many ways in which we can improve the models