From Reactions to Observations: the Directed Bigraphical Model

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Distributed and concurrent computational systems are naturally represented as *reaction systems* (RSs), where the dynamics is represented by a reduction relation on the possible configurations. Despite their evident simplicity, reduction systems are not very apt to support verification and analysis of computational systems, due to their intensional and non-compositional nature. In fact, most widespread and developed reasoning methodologies are based on *behavioral* models, which make explicit the properties to be analysed. In this respect, *labelled transition systems* (LTS) are particularly important: an LTS specifies the interaction capabilities of each component by means of *labels*, representing what can be *observed* by the environment. Many methodologies for proving behavioral equivalences and verifying properties, including *bisimulations* and *modal/temporal logics*, rely on this kind of specification.

Usually, LTSs are crafted "by hand", but the more complex is the calculus, the more difficult is to devise an adequate LTS. In particular, the induced bisimilarity should be a congruence in order to allow compositional reasoning over the systems. For this reason, much work has been spent recently in looking for general methodologies for deriving LTSs from reduction systems. More generally, Sewell argued that the labels ℓ for an agent a are all contexts (i.e., "agents with a hole") $\ell[\cdot]$ such that $\ell[a]$ triggers a reaction. Remarkably, the bisimulation induced by this LTS is always a congruence. However, this LTS is also very large, because its labels are closed under composition with all possible contexts: if $\ell[\cdot]$ is a label for a then for any other context $d[\cdot]$, $d\ell[\cdot]$ is a label as well.

Leifer and Milner have observed that these redundant transitions can be drastically reduced by restricting to "minimal" labels, that is, contexts which are not obtained by composition. The minimality condition can be expressed by the categorical notion of *relative pushout* (RPO). RPOs do not always exist but, if the agents and contexts of a reactive system are arrows of a category where RPOs can be calculated, we can readily derive a labelled transition system much smaller than the full one and whose bisimulation is still sound and compositional. Hence, in order to apply this theory, the problem is to find the categories where the calculi and systems used in Concurrency can be conveniently represented, and RPOs can be constructed.

To this end, an emerging meta-model are Milner's *bigraphical reactive systems*, which are a graphical model of computation where both *locality* and *connectivity* are prominent. The configuration of the system is represented by a *bigraph*, which is composed by two orthogonal structures: a hierarchical *place graph* describing locations, and a *link (hyper-)graph* describing connections.

We develop a bigraphical meta-model, called *directed bigraphs*, which subsumes and generalizes both previous kinds of bigraphs and respective RPO constructions. Directed bigraphs allow to represent uniformly a wider range of computational models within a single theory, and offers new, original features not available in previous meta-models.

An intuitive explanation of directed bigraphs is to notice that names are not resources on their own, but only denote, or give access to, (abstract) resources (represented by "edges"). Names are exposed on the interfaces of systems, much like "ports". If a name on an interface is associated to a resource of the system, then the system is *offering* the resource to the outside; on the converse, a name not associated to any resource can be seen as a formal parameter of the system, that is the system is *asking* for an external resource through that name. Thus, we can discern a "resource request flow" which starts from controls, goes through names on interfaces, and eventually terminates in edges.

Directed bigraphs can be used for defining reactive systems, henceforth called *directed bigraphical reactive system* (DBRS). Given a DBRS, following the RPO construction we can readily obtain a labelled transition system (called *directed bigraphical transition system*, DBTS) by taking as labels the "minimal" contexts identified by RPOs. We show that the bisimilarity associated to this DBTS is always a congruence.