Linear Programming Models for Traffic Engineering Under Combined IS-IS and MPLS-TE Protocols

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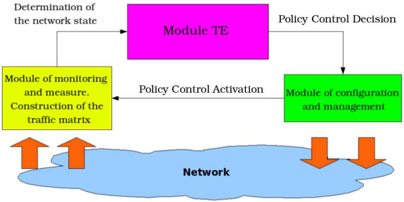
Part I

Introduction and Background

Problem Description

Scenario

- Very large scale networks have been built by the Network Engineers
- Experience and Best Common Practice
  - Planning
  - Reaction to critical Network Events



Network survivability Techniques

- Network Design and Capacity Allocation
- Traffic Management and Restoration

Autonomous System (AS)

Collection of IP Networks and routers controlled by a single administrative entity

Two routing protocols

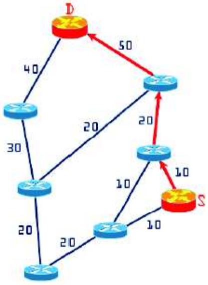
- End System-to-Intermediate System (ES-IS)
- Intermediate System-to-Intermediate System (IS-IS)

IS-IS: link state routing protocol

Interior Gateway Protocol

IS-IS/OSPF

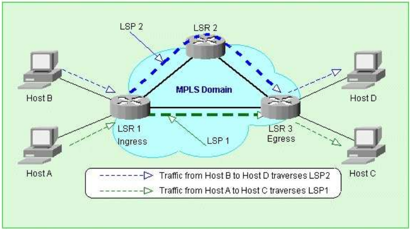
- Metric associated to each arc
- Route selection using Dijkstra's Shortest Path Algorithm
- Equal Cost Multiple Paths (ECMP)



MPLS Technology

MPLS-TE

- Allows the configuration of the traffic in order to optimize the resources.
    - Allows the building of VPN (Virtual Private Networks), using **LSP (Label Switched Paths)-Tunnels**.
- Extends existing IP protocol



Restoration Schemes: Link Restoration

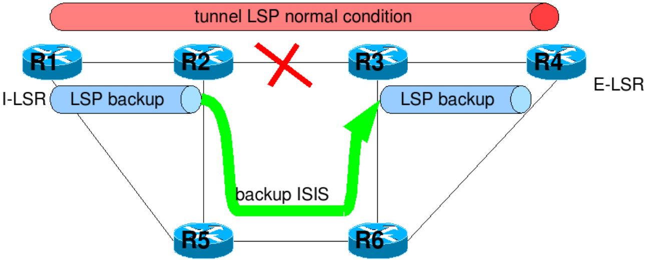


Figure: Link Restoration for single failure condition

Restoration Schemes: Path Restoration

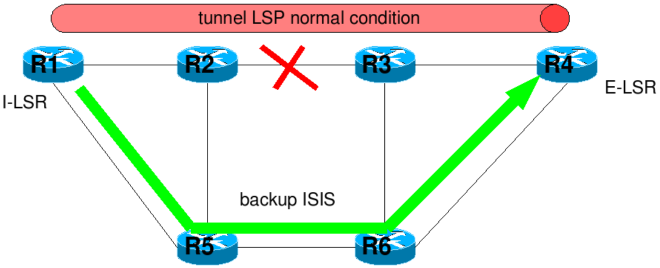


Figure: Path Restoration for single failure condition

Failure Analysis

- 20% : scheduled network maintenance activities
- 80% : unplanned failures where :
  - 30% shared link failures
  - 70% single link failures

Problem Statement

- Is it possible to obtain a robust configuration of the network using the combination of IS-IS routing and MPLS-TE techniques?
- Is it possible to formulate the question as a pure LP problem?

Linear Programmin Models

- $\min c \cdot x$
- $A \cdot x = b$
- $x \geq 0$

Graphs and Network Flows

- Generally, in Operations Research, the term network denotes a weighted graph $G = (N, A)$ where the weights are numeric values associated to nodes and/or arcs of the graph.

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MMCF Problem Definition

Notation

- $G = (N, A)$ where N is the set of nodes and $A \subseteq N \times N$ is the set of arcs
- K is the set of commodities
- $h - th$ commodity determined by: (d^h, s^h, t^h), where $s^h \in N$ and $t^h \in N$, with $s^h \neq t^h$, are the starting and ending node, and d^h is the quantity to be moved from s^h to t^h

Formulations

- node-arc formulation : the variables are x_{ij}^h
- arc-path formulation: the variables are f_p

MMCF: Node-arc formulation

The problem

- $\min \sum_{h \in \mathcal{K}} \sum_{(i,j) \in \mathcal{A}} c_{ij}^h \cdot x_{ij}^h$
- $\sum_{(j,i) \in \mathcal{BS}(i)} x_{ji}^h - \sum_{(i,j) \in \mathcal{FS}(i)} x_{ij}^h = \begin{cases} -d^h & i = s^h \\ d^h & i = t^h \\ 0 & \text{otherwise} \end{cases}$
- $\sum_{h \in K} x_{ij}^h \leq u_{ij} \quad (i,j) \in A$
- $x_{ij}^h \geq 0 \quad (i,j) \in A$

$|N||K| + |A|$ constraints
$|K||A|$ variables

MMCF: Arc-path formulation

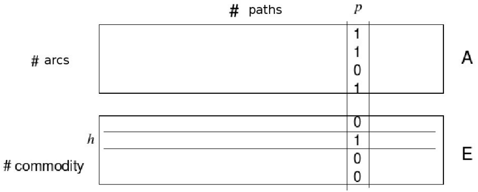
The notation

- P^h: the set of paths in G from the node s^h to the node t^h
- $P = \cup_{h \in K} P^h$: the set of all relevant paths
- $p \in P$ belongs to a unique commodity, identified by the starting and ending nodes of p; $h(p)$
- $c_p = \sum_{(i,j) \in p} c_{ij}$ cost of the path p

The problem

- $\min \sum_{h \in \mathcal{K}} \sum_{p \in \mathcal{P}^h} c_p$
- $\sum_{p \in P^h} f_p \quad = d^h \quad h \in K$
- $\sum_{p:(i,j) \in p} f_p \quad \leq u_{ij} \quad (i,j) \in A$
- $f_p \quad \geq 0 \quad p \in P$

The problem has $|A| + |K|$ constraints



Column generation.2

Let's consider a reduced set of paths $B \subset P$.

A_B	$A_{(P/B)}$
E_B	$E_{(P/B)}$

P_B

- $\min c_B f_B$
- $A_B f_B \quad \leq u$
- $E_B f_B \quad = d$
- $f_B \quad \geq 0$

D_B

- $\max \quad \lambda u + \gamma d$
- $\lambda A_B + \gamma E_B \quad \leq c_B$
- $\lambda \quad \leq 0$

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- If we solve the master problem we obtain: $\hat{f}_B$ and $(\hat{\lambda}, \hat{\gamma})$.
- Strong Duality: if $(\hat{\lambda}, \hat{\gamma})$ is feasible to DMMCF then $\hat{f}_B$ is optimal for MMCF
- Feasibility of the dual problem can be conveniently restated in terms of *reduced cost* of paths

$$\bar{c}_p = c_p - \hat{\gamma}_{h(p)} - \sum_{(i,j) \in P} \hat{\lambda}_{ij} = \sum_{(i,j) \in P} (c_{ij} - \hat{\lambda}_{ij}) - \hat{\gamma}_{h(p)} \quad .$$

- For all the paths p that belong to the set B we have that $\bar{c}_p \geq 0$.
- What can we say about the paths $p \in P \setminus B$?

Column generation.4

- For each $h \in K$, we compute a minimum cost path from s^h to t^h by associating with each arc the new costs $c_{ij} - \hat{\lambda}_{ij}$
- This minimum cost path is indicated by $\hat{p}_h$ *reduced cost* of paths
- We compute the *reduced cost* $\bar{c}_{\hat{p}_h}$: if it's greater or equal to zero for all $h \in K$: the set B holds the optimal paths
- If, instead, at least one path $\hat{p}_h$ has negative reduced cost, then it can be added to B and the process is iterated.

Part II

Models

LP MODEL with Node-Arc Formulation

Data

- $\mathcal{N}$ - Node set
- $\mathcal{A}$ - Edge set
- $\mathcal{F}$ - Commodity set
- u_{ij} - Capacity associated with link (i, j)
- d^f - Effective bit rate of flow f
- x_{ij}^f - Share of flow f carried by IS-IS and traversing link (i, j)

Variables

- u_{max} - Maximum utilization in the network - objective function
- is^f - Flow f carried by IS-IS
- $flow_{ij}^f$ - Flow f carried by MPLS and traversing link (i, j)

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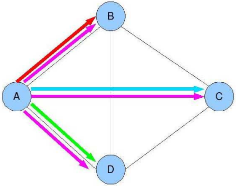
Flows Aggregation

Commodities aggregation by source node

$$flow_{ij}^{h} = \sum_{f:I(f)=h} flow_{ij}^{f}$$



Example

Commodities $A \rightarrow B$, $A \rightarrow C$, and $A \rightarrow D$ are replaced by a single commodity "A"

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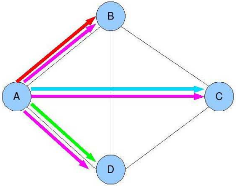

Example

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General Routing Problem

Objective function

$z = \min(u_{max})$

$\sum_{f \in \mathcal{F}} x_{ij}^f \cdot is^f + \sum_{h \in \mathcal{N}} flow_{ij}^h \leq u_{max} \cdot u_{ij} \quad \forall (i,j) \in \mathcal{A}$

$$\sum_{j:(j,i) \in \mathcal{A}} flow_{ji}^h - \sum_{j:(i,j) \in \mathcal{A}} flow_{ij}^h = \begin{cases} -\sum_{f \in F(h)} d^f + is^f & i = h \\ d^f - is^f & \text{if } i \neq h, i = E(f), f \in F(h) \\ 0 & \text{otherwise} \end{cases}$$

$flow_{ij}^h \geq 0 \quad \forall (i,j) \in \mathcal{A}, \ \forall h \in \mathcal{N}$

$is^f \geq 0 \quad \forall f \in \mathcal{F}$

General Routing Problem

Capacity constraints

$z = \min(u_{max})$

$\sum_{f \in \mathcal{F}} x_{ij}^f \cdot is^f + \sum_{h \in \mathcal{N}} flow_{ij}^h \leq u_{max} \cdot u_{ij} \quad \forall (i,j) \in \mathcal{A}$

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Flow conservation equations

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General Routing Problem

Positivity constraint

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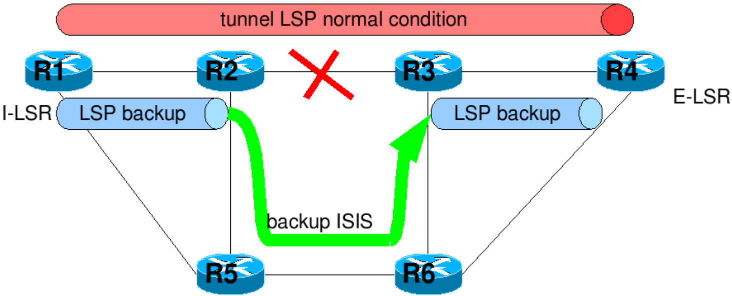
Link restoration



Survivability Constraints

$$\sum_{f \in \mathcal{F}} x_{ij}^{f,l} \cdot is^f + \sum_{h \in \mathcal{N}} flow_{ij}^h + \sum_{h \in \mathcal{N}} (x_{ij}^{l_+,l} \cdot flow_{l_+}^h + x_{ij}^{l_-,l} \cdot flow_{l_-}^h) \leq u_{max} \cdot c_{ij} \quad \forall (i,j) \neq l_+, l_- \in \mathcal{A}$$

Share of flow carried by IS-IS when edge l fails

Flow carried by explicit MPLS LSP along link (i,j)

Share of flow flowing through edge l from node p to node q (arc l_+) and those from node q to node p (arc l_-) that is rerouted by IS-IS along link (i,j)

Proof

Survivability Constraints

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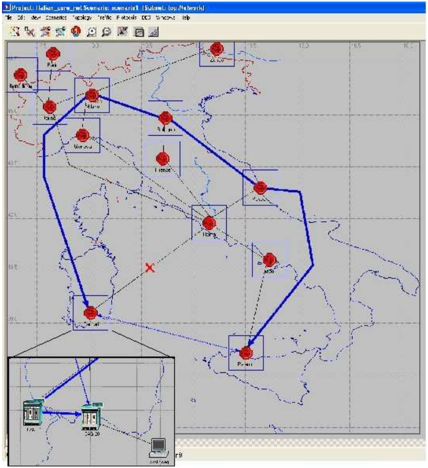
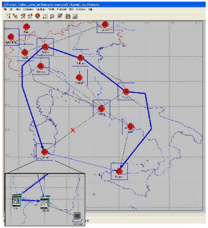
▸ Proof

TINet Italy-Normal Condition



- 18 nodes
- 54 arcs
- 306 flows
- 1279 variables
- 378 constraints

TINet Italy-Normal Condition



- 18 nodes
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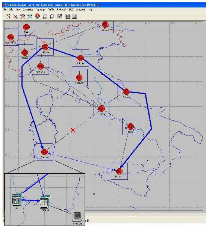
- 1279 variables
- 378 constraints

Routing Optimization

	u_{max}	Gain (Def)	Gain (Tis)	# LSP
Default	$u_{max} = 72\%$	–	–	0
Existing metrics	$u_{max} = 66\%$	8.3%	–	0
IS-IS opt.	$u_{max} = 61\%$	15.2%	7.6%	0
MPLS-TE opt.	$u_{max} = 59\%$	18.1%	10.6%	105

TINet Italy - Survivability

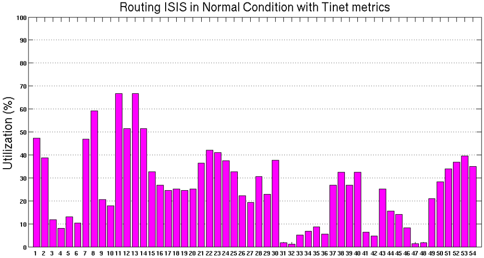


- 18 nodes
- 54 arcs
- 306 flows
- 1279 variables
- 1782 constraints

Survivability Optimization

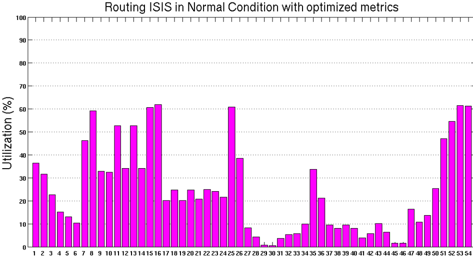
	u_{max}	Gain (Def)	Gain (Tis)	# LSP
Default	$u_{max} = 128\%$	–	–	0
Existing metrics	$u_{max} = 117\%$	8.6%	–	0
IS-IS opt.	$u_{max} = 85\%$	33.6%	27.3%	0
MPLS-TE opt.	$u_{max} = 83\%$	35.2%	29.1%	86

Graphical Results - Routing



Statistics

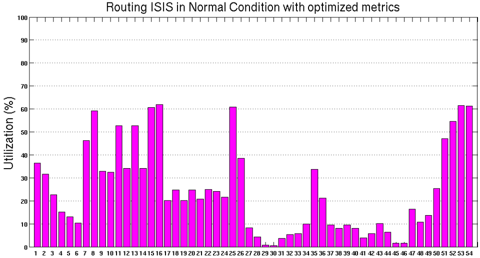
$u_{max} = 66\%$
Average $= 26.12\%$
Variance $= 0.028$



Statistics

$u_{max} = 61\%$
Average $= 24.52\%$
Variance $= 0.037$

Graphical Results - Routing



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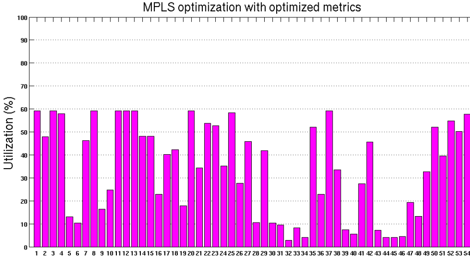
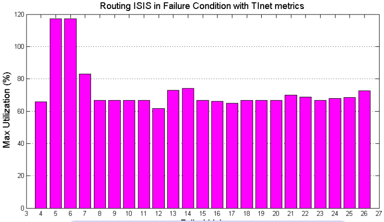


Statistics

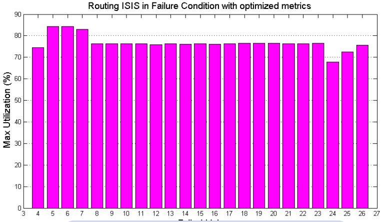
$u_{max} = 59\%$
Average $= 27.22\%$
Variance $= 0.029$

Graphical Results - Survivability



Statistics

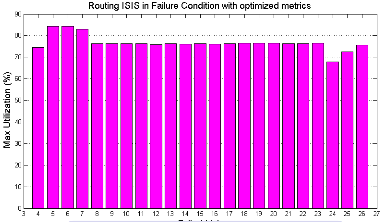
$u_{max} = 117\%$
Average $= 72.66\%$
Variance $= 0.021$

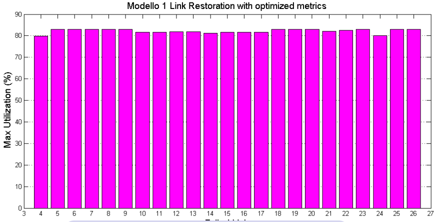


Statistics

$u_{max} = 85\%$
Average $= 76.63\%$
Variance $= 0.001$

Graphical Results - Survivability





Statistics

$u_{max} = 85\%$
Average $= 76.63\%$
Variance $= 0.001$

Statistics

$u_{max} = 83\%$
Average $= 81.59\%$
Variance $= 0.0002$

IBCN European Network

- 37 nodes
- 114 arcs
- 1332 flows
- 5551 variables
- 1483 constraints
- 7867 constraints (with survivability)

IBCN - Normal condition

- 37 nodes
- 114 arcs
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- 7867 constraints (with survivability)

Routing Optimization

	Work. Cond.	Failure Cond.	# LSP
IS-IS/OSPF with def. metrics	71%	101%	0
IS-IS with optim. metrics	54%	74%	0
LP models with optim. metrics	40%	64%	543

IBCN - Graphical Results

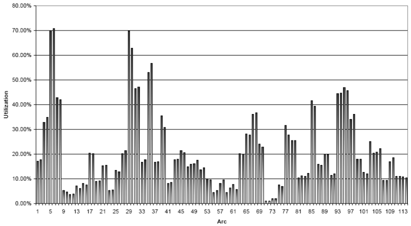


Figure: *Is-Is Routing Normal Condition Default Metrics* **Umax=71%**

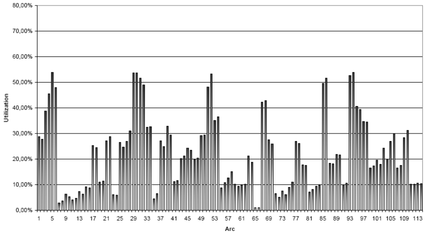


Figure: *Is-Is Routing Normal Condition Optimized Metrics* **Umax=54%**

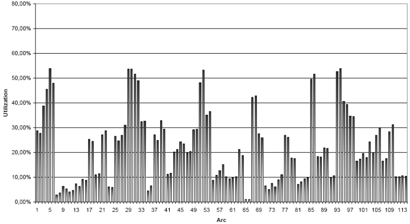


Figure: *Is-Is Routing Normal Condition Optimized Metrics* **Umax=54%**

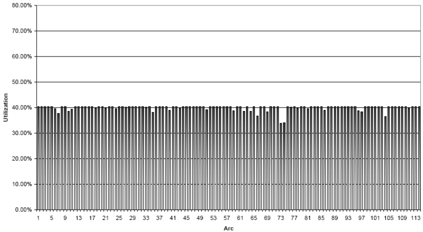


Figure: *Mpls Routing Normal Condition Optimized Metrics* **Umax=40%**

IBCN - Graphical Results

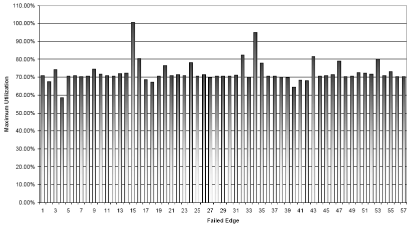


Figure: *Is-Is Routing Failure Condition Default Metrics* **Umax=101%**

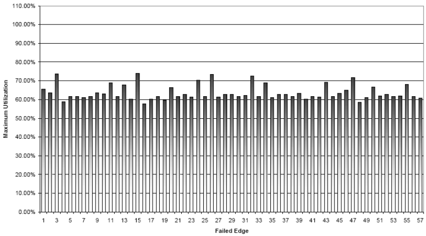


Figure: *Is-Is Routing Failure Condition Optimized Metrics* **Umax=74%**

IBCN - Graphical Results

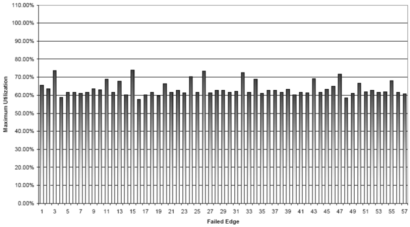


Figure: *Is-Is Routing Failure Condition Optimized Metrics* **Umax=74%**

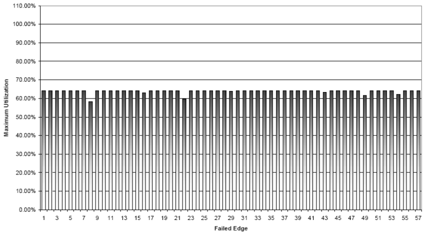


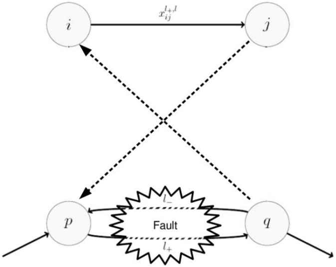
Figure: *Mpls Routing Failure Condition Optimized Metrics* **Umax=64%**

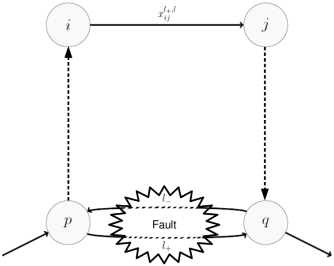
An extended description of this work is available as Technical Report of the University of Pisa at the following link:

http://compass2.di.unipi.it/TR/Files/TR-08-24.pdf.gz

Thank you for your attention

Survivability Constraints





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