Coalgebraic Logics for Knowledge Representation and Reactive Systems

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University of Leicester, June 24, 2011

Introduction: Modal Logic in Computer Science

- Description logics
  - Core formalism of KR and the Semantic Web
  - Underlying logic of OWL-DL
- Temporal logics (CTL, LTL)
- (and many more: epistemic, deontic, . . .)
- Relational semantics
  - Binary relations between individuals
  - Guarded universal and existential quantification

Beyond Relational Semantics

Many modes of expression need more than relational semantics, e.g.

Uncertainty	(Probabilities)
Vagueness	(Fuzzy truth values)
Defeasibility	(Preference orderings)
Causation and agency	(Games)

Large variety of domain-specific logics

+ Suitable expressive means for every purpose

– Multiplied need for tools and algorithms

Enter Coalgebra

Sichere Kognitive Systeme

Coalgebra acts as a unified framework for real-life reasoning

- semantically
- logically
  - generic complete axiomatizations
- algorithmically
  - generic decidability results
  - generic algorithms and complexity analysis

Overview

- Real-life reasoning
- Review of relational semantics
- Coalgebraic logic
- One-step rules and generic algorithms

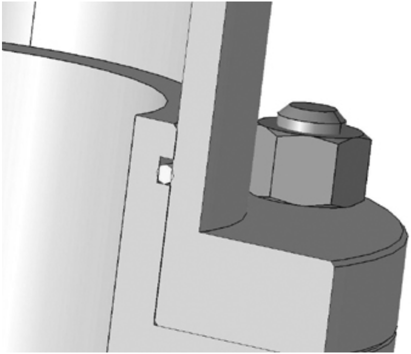
OWL in CAD Quality Control

- CATIA DMU Analyser:
  - Overlaps of parts

OWL in CAD Quality Control

- CATIA DMU Analyser:
  - Overlaps of parts
  - Not every overlap is an error
- OWL Ontology:

$$\text{part} \sqsubseteq \text{overlaps only gasket}$$
$$\sqcup\ (\text{bolt} \sqcap \text{overlaps only nut})$$
$$\sqcup \ldots$$

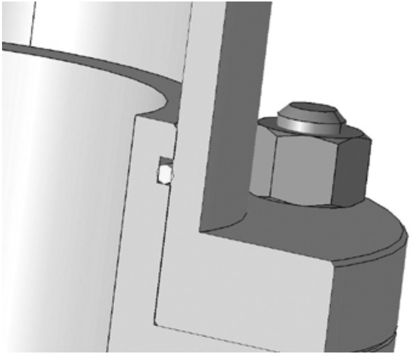


(Franke/Klein/Schröder/Thoben CIRP Design 2010)

Conditional logic in CAD Quality Control

$a \Rightarrow b$:
If a then normally b.

part $\Rightarrow$ overlaps only nothing
gasket $\Rightarrow$ overlaps some part
bolt $\Rightarrow$ overlaps some nut
bolt $\sqcap$ hasExplicitPart some thread
$\Rightarrow$ overlaps only nothing



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OWL in the Representation of CPGs

Sichere Kognitive Systeme

[*BMBF KMU Innovativ* project SIMPLE
Semantically founded implementation of clinical practice guidelines]

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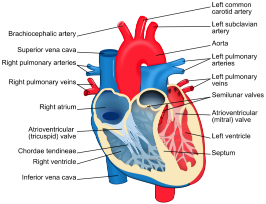
> ***7-13*** *In presence of medium prior probability and inconclusive ergometry, an exercise test with imaging should be carried out.*



Approximation in relational DL:

$$\forall \mathsf{hasPriorRiskCHG}.\,\mathsf{Medium} \sqcap \forall \mathsf{hasDiagnostics}.\,\neg\mathsf{Ergometry} \sqcup \mathsf{Inconclusive} \sqsubseteq \exists \mathsf{hasRecommendedDiagnostics}.\,(\mathsf{ExerciseTest} \sqcap \exists \mathsf{hasObservation}.\,\mathsf{Imaging})$$

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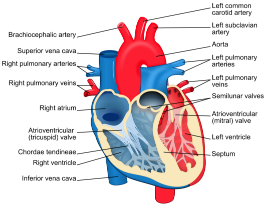

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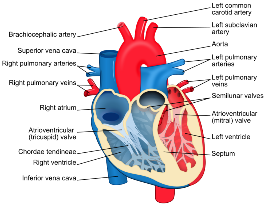

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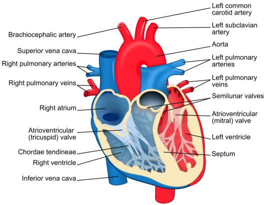

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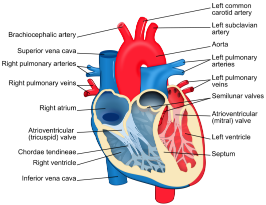


Better approximation in coalgebraic description logic:

$$\text{moderately}(\text{probably}\ (\exists.\,\text{hasDisorder}.\,\text{CHD}))\sqcap \forall\text{hasDiagnostics}.\,\neg\text{Ergometry}\sqcup\text{Inconclusive} \Rightarrow \exists\text{hasRecommendedDiagnostics}.\,(\text{ExerciseTest}\sqcap\exists\text{hasObservation}.\,\text{Imaging})$$

More examples from CPGs

Nested defeasible implication:

Units normally seeing at least 100 new cases of cancer per annum should be able to maintain their expertise.

Comparison of probabilities:

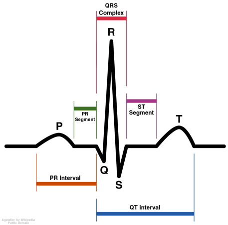
Radiotherapy should be given following mastectomy or breast conserving surgery [. . .] where the benefit to the individual is likely to outweigh risks of radiation related morbidity.

(SIGN breast cancer CPG)

Combined vague temporality, belief, and uncertainty:

Aspirin should be given to all patients with a STEMI as soon as possible after the diagnosis is deemed probable.

(European CPG for acute ST-segment elevated myocardial infarction)



Relational Semantics of DL

Concepts

$$C ::= \bot \mid A \mid \neg C \mid C_1 \sqcap C_2 \mid \forall R.\, C$$

Interpretations $\mathcal{I}$:

- $(\Delta^{\mathcal{I}}, (A^{\mathcal{I}}), (R^{\mathcal{I}}))$ where
  - $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
  - $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- Extension $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ of concepts C:

$$(\forall R.\, C)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} \mid \forall y \in \Delta^{\mathcal{I}}.\, xR^{\mathcal{I}}y \Rightarrow y \in C^{\mathcal{I}}\}$$

E.g.

$$\mathsf{ChessFanatic} = \mathsf{ChessPlayer} \sqcap \forall \mathsf{hasFriend}.\, \mathsf{ChessFanatic}$$

Incomplete Overview of Non-relational Logics

Logic	Systems	Syntax	Reading
Probabilistic logics	Markov chains	$L_p C$	With Prob. $\geq p$, C
Graded logics	Multigraphs	$\geq nR.\,C$	$\geq n$ R-successors satisfy C
Conditional logics	Preference models	$C \Rightarrow D$	If C then normally D
Alternating-time logic	Concurrent game struct.	$\langle\langle C \rangle\rangle C$	Coalition C can force C
Game logic	Game models	$\langle \gamma \rangle C$	Angel can force C in game γ

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Reactive systems

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Multi-agent systems

Tool Support

…for such logics has been notoriously limited:

- CondLean: weak conditional logics
- Pronto: $P\text{-}\mathcal{SHIQ}(D)$.

A Reformulation of Relational Semantics

- Interpretations of role R are $\mathcal{P}$-coalgebras

$$\xi_R : \Delta^{\mathcal{I}} \to \underbrace{\mathcal{P}}_{\text{functor}}(\Delta^{\mathcal{I}})$$

- Extension of $\forall R.\, C$:

$$(\forall R.\, C)^{\mathcal{I}} = \{x \in \Delta_I \mid \xi_R(x) \in \underbrace{\{A \in \mathcal{P}(\Delta^{\mathcal{I}}) \mid A \subseteq C^{\mathcal{I}}\}}_{=: \underbrace{[\![\forall R]\!]_{\Delta^{\mathcal{I}}}}_{\text{predicate lifting}}(C^{\mathcal{I}})}\}$$

Coalgebraic Logic

- General modal signatures Σ
  (sets of finitary modal operators)
- Abstraction of the type of interpretations:
  - Functor (parametrized data type) $T : \mathbf{Set} \to \mathbf{Set}$
  - Interpretations = T-coalgebras

$$\xi : \Delta^{\mathcal{I}} \to T(\Delta^{\mathcal{I}})$$

- Abstraction of the semantics of operators $L \in \Sigma$:
  - predicate liftings $[\![L]\!]_X : \mathcal{P}(X) \to \mathcal{P}(TX)$, natural in X
  - $(LC)^{\mathcal{I}} = \xi^{-1}[[\![L]\!]_{\Delta^{\mathcal{I}}}(C^{\mathcal{I}})]$

(Pattinson 2003, Schröder 2005)

Nearly Everything is Coalgebraic

Logic	Systems	Syntax	Functor
Classical DLs	Relational models	$\forall R.\,C$	Powerset $\mathcal{P}(X)$
Probabilistic logics	Markov chains	$L_p C$ $\sum a_i P(C_i) \geq b$	Distributions $D(X)$
Graded logics	Multigraphs	$\geq nR.\,C$ $\sum a_i \#(C_i) \geq b$	Multisets $\mathcal{B}(X) = X \to \mathbb{N}_\infty$
Conditional logics	Preference models	$C \Rightarrow D$	Preference orders $\exists(S, \preceq).S \to X$
Alternating-time logic	Concurrent game struct.	$[c]C$	Games $\exists(S_i).(\prod S_i \to X)$
Game logic	Game models	$\langle\gamma\rangle C$	Upclosed nbhd. systems

(Schröder/Pattinson/Cirstea/Kurz/Venema et al. 2004–2010)

Example: Local Type-1 Probabilistic Logic

(Fagin/Halpern JACM 1994)

Functor $D(X)$ = distributions on X

Interpretations $\Delta^{\mathcal{I}} \to D(\Delta^{\mathcal{I}})$ = Markov chains

Operators L_p 'with probability $\geq p$'

$$[\![L_p]\!]_X(A) = \{\mu \in D(X) \mid \mu(A) \geq p\}$$

Example: Alternating Time

(Alur et al. JACM 2002)

$N = \{1, \dots, n\}$ set of agents, $c \subseteq N$ coalition

Functor:

$$F(X) = \Big\{(k_1, \dots, k_n, f) \mid f : \big(\prod_{i \in N}\{1, \dots, k_i\}\big) \to X\Big\}$$

Interpretations $\Delta^{\mathcal{I}} \to F(\Delta^{\mathcal{I}})$ = concurrent game structures

Operators $[c]$ 'c can force … in the next step'

$$[\![[c]]\!]_X(A) = \{f \in F(X) \mid \exists \sigma_c. \forall \sigma_{N-c}. f(\sigma_c, \sigma_{N-c}) \in A\}$$

Generic Deduction Systems

Parametrized Systems:

- Fixed propositional part
- Further fixed parts depending on orthogonal features (nominals, fixed points)
- Parameter: Axiomatization of the functor through (cutfree complete) one-step rules
  (Schröder/Pattinson LICS 06; see my Leicester seminar talk of March 2006)

One-Step Rules

One-step logic: V set of prop. var.,

$$\Sigma V = \{La \mid a \in V, L \in \Sigma\}.$$

Given $\tau : V \to \mathcal{P}(X)$, interpret

- propositional formulas φ over V as $[\![\varphi]\!]\tau \subseteq X$
- propositional formulas ψ over ΣV as $[\![\psi]\!]\tau \subseteq TX$ by

$$[\![La]\!]\tau = [\![L]\!]_X \tau(a)$$

One-step rules: $\dfrac{\varphi}{\psi}$ — φ propositional over V, ψ clause over ΣV

$\dfrac{\varphi}{\psi}$ one-step sound if $[\![\varphi]\!]\tau = X \implies [\![\psi]\!]\tau = TX$.

The Cut Rule

$$\frac{A \to C \qquad C \to B}{A \to B}$$

Undesirable for proof search.

One-Step Cut-Free Completeness

Set $\mathcal{R}$ of one-step rules one-step cut-free complete
if for clauses χ over ΣV

$$[\![\chi]\!]\tau = TX \implies \exists \varphi/\psi \in \mathcal{R}, \sigma : V \to V.$$
$$[\![\varphi\sigma]\!]\tau = X, \quad \psi\sigma \text{ contracted}, \quad \psi\sigma \subseteq \chi.$$

$$\frac{\dfrac{\varphi\sigma}{\psi\sigma}}{\chi}$$

One-step cut-free complete rule sets (OSCCR)

- induce tableau-based model constructions
- yield cut-free complete deduction systems for the full logic
  $\to$ proof search

One-Step Rules: Examples

$\mathcal{ALC}$:

$$\frac{\bigsqcup_{i=1}^{n} \neg a_i \sqcup b}{\bigsqcup_{i=1}^{n} \neg \forall R.\, a_i \sqcup \forall R.\, b} \quad (n \geq 0)$$

Local type-1 probabilistic logic:

$$\frac{\overbrace{\sum_{i=0}^{n} r_i a_i \succ \sum_{i=0}^{n} r_i p_i}^{\text{Arithmetic of characteristic functions}}}{\bigsqcup_{0 \leq i \leq n} sgn(r_i) L_{p_i} a_i}$$

where $n \geq 0$, $r_i \in \mathbb{Z} - \{0\}$, $\succ = \begin{cases} > & \text{if } r_i < 0 \text{ for all } i \\ \geq & \text{otherwise} \end{cases}$

Generic Algorithms via OSCC Rule Sets

Sichere Kognitive Systeme

- PSPACE for next-step-logics
- PSPACE for coalgebraic hybrid logic
- EXPTIME for coalgebraic description logics (i.e. with TBoxes)
- Completeness and EXPTIME global caching for flat fixed point logics via $\mathcal{O}$-adjointness (Schröder/Venema 2010)
  - Alternating μ-calculus (Alur et al. 2002)
  - Graded μ-calculus (Kupferman et al. 2002)

Flat Coalgebraic Fixed Point Logics

Flat fixed point operators

$$\sharp_\gamma(\varphi) \equiv \mu x.\, \gamma(\varphi, x)$$
$$\flat_\gamma(\varphi) \equiv \nu x.\, \gamma(\varphi, x) \qquad (\gamma \text{ modal})$$

$\to$ fragments of single-variable coalgebraic μ-calculus.

E.g.

- CTL: $AF\varphi = \sharp_{p\vee\Box x}\varphi$
- $\flat_{p\wedge\Box\Box x}$ not in CTL*
- ATL: $\langle\langle C\rangle\rangle F\varphi = \sharp_{p\vee[C]x}\varphi$
- Graded μ-calculus (Kupferman et al. 2002):

$$\sharp_{p\vee\Diamond_2 x}\varphi$$

'the current state is the root of a binary tree whose leaves satisfy φ'.

The Kozen-Park Axioms

Briefly: '$\sharp_\gamma(\varphi)$ is a least fixed point', i.e.:

Unfolding:

$$\gamma(\varphi, \sharp_\gamma\varphi) \to \sharp_\gamma\varphi$$

Fixed-point induction:

$$\frac{\gamma(\varphi, \chi) \to \chi}{\sharp_\gamma(\varphi) \to \chi}$$

Are these complete?

- Do imply that $\sharp_\gamma(\varphi)$ is a least fixed point in the Lindenbaum algebra

Strategy for the Completeness Proof

- Show constructivity of the Lindenbaum algebra:

$$\sharp_\gamma(\varphi) = \bigvee_{i<\omega} \gamma(\varphi)^i(\bot)$$

via $\mathcal{O}$-adjointness of $\gamma(\varphi)$: for all ψ there is a finite set $G_{\gamma(\varphi)}(\psi)$ s.t.

$$\gamma(\varphi,\rho) \leq \psi \iff \rho \leq \chi \quad \text{for some } \chi \in G_{\gamma(\varphi)}(\psi)$$

- Constructivity implies

$$\sharp_\gamma\varphi \wedge \psi \quad \text{consistent} \implies \gamma(\varphi)^i(\bot) \wedge \psi \quad \text{consistent for some } i < \omega.$$

- Tableau construction with time-outs

$\mathcal{O}$-Adjointness via OSCC Rule Sets

- Unfolding & guardedness:
  w.l.o.g. the top level of every formula is modal
- Rigidity lemma:
  w.l.o.g. proofs of modal clauses end in modal one-step rules

Example: Adjointness of $\Box$. Recall rule:

$$\frac{\bigwedge_{i=1}^{n} a_i \to b}{\bigwedge_{i=1}^{n} \Box a_i \to \Box b} \quad (n \geq 0)$$

Calculate:

$$\Box\rho \leq \psi = \bigwedge_{i=1}^{n} \Box\chi_i \to \bigvee_{j=1}^{m} \Box\theta_j$$

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Thus put $G_{\Box x}(\psi) = \{\bigwedge_{i=1}^{n} \chi_i \to \theta_j \mid j = 1, \dots, m\}$

Conclusions

- Coalgebra provides a uniform framework for modal and hybrid logics
  - Graded operators (knowledge representation, redundancy)
  - Probabilistic operators (quantitative uncertainty, reactive systems)
  - Conditional operators (nonmonotonic reasoning)
  - Alternating-time logics, game logic, logics of agency (multi-agent systems)
- Wide range of generic decision procedures and complexity bounds
- Modular (Schröder/Pattinson ICALP 2007)
- Frequently new bounds and calculi for instance logics, in particular in presence of
  - nominals
  - fixed points

Ongoing and Future Work

- Manydimensional coalgebraic logics
- Fuzzy coalgebraic logics
  - E.g. the logic of probably
- Vision: generic, efficient modular reasoning tools
  - Ongoing optimization of CoLoSS (PhD thesis Hausmann)
  - Enable use in realistic applications, e.g. CPGs

Thanks for your attention!

CDL with Nominals

- Nominals $i, j, \ldots$ are atomic concepts to be interpreted as singletons
- Internalize ABoxes via satisfaction operators

$$@_i C = \text{`}i \text{ satisfies } C\text{'}$$

Deduction over GCIs

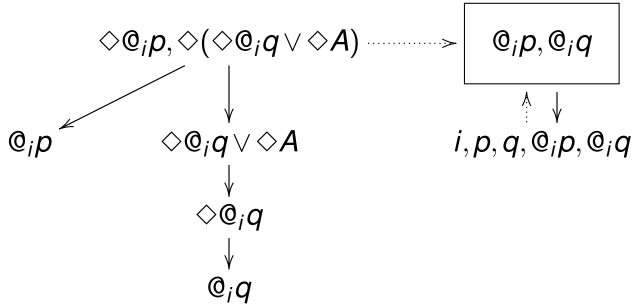
- General concept inclusions $C \sqsubseteq D$
- Tableaux diverge without blocking:
  for gci $\top \sqsubseteq \exists R.\, A$,

$$\cfrac{\top, \exists R.\, A}{\cfrac{A, \exists R.\, A}{\cfrac{A, \exists R.\, A}{\dots}}}$$

- Tedious analysis even for $\mathcal{ALC}$ (Donini/Massacci 99)

A Global Caching Algorithm

Collect @-formulas along a winning strategy:



- Decidability in EXPTIME
- Room for heuristic optimization
- Novel algorithm even for the relational case

(Goré/Kupke/Pattinson/Schröder IJCAR 10)