JUDGMENT AGGREGATION TUTORIAL - PART II

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SOME CURRENT TOPICS IN JUDGMENT AGGREGATION

THE TWO PHASES OF JA

It's impossible!

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It's impossible!

Yes, we can!

SOME CURRENT TOPICS (1)

(A short, incomplete and biased list)

- Computational complexity
- Definition and study of feasible aggregation rules
- Argumentation

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SOME CURRENT TOPICS (2)

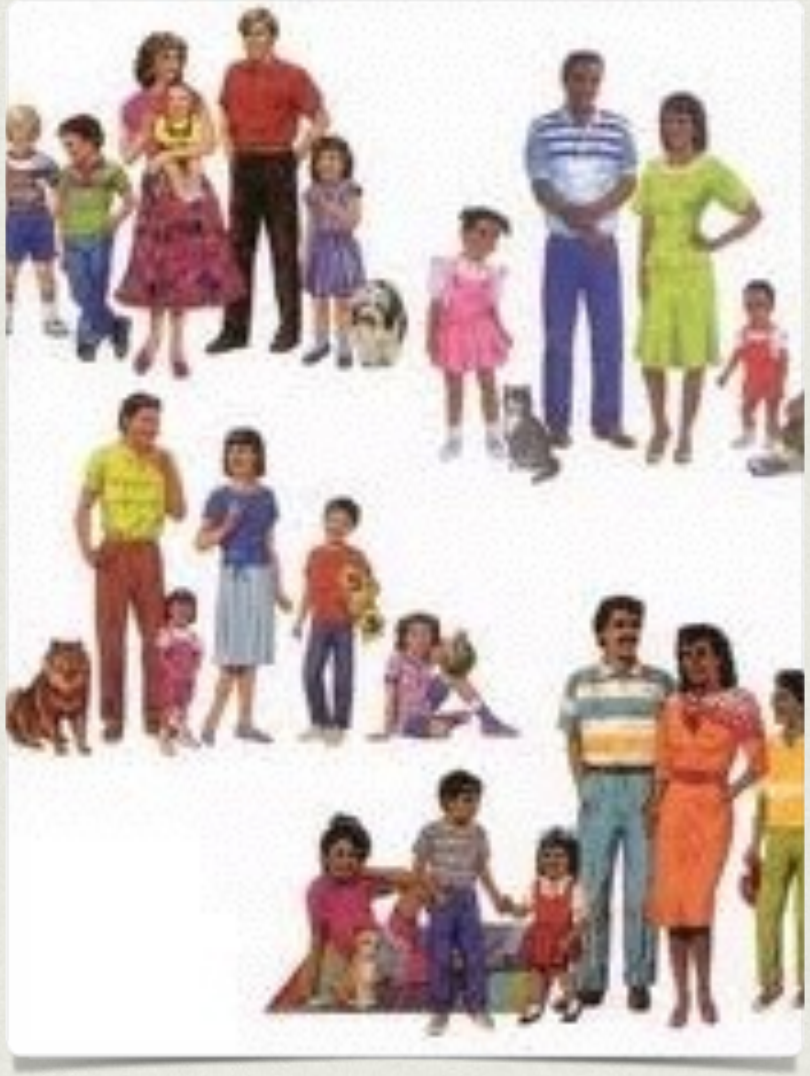
- Computational social choice: a field stemming from the interaction between computer science and social choice theory, which studies:
  - the complexity of the *application* of aggregation rules
  - the complexity of manipulating aggregation rules
  - the design of aggregation rules based on knowledge representation techniques (e.g. belief merging)
  - etc.

SOME CURRENT TOPICS (2)

- Computational social choice: a field stemming from the interaction between computer science and social choice theory, which studies:
  - the complexity of the *application* of aggregation rules
  - the complexity of manipulating aggregation rules
  - the design of aggregation rules based on knowledge representation techniques (e.g. merging)
  - etc.

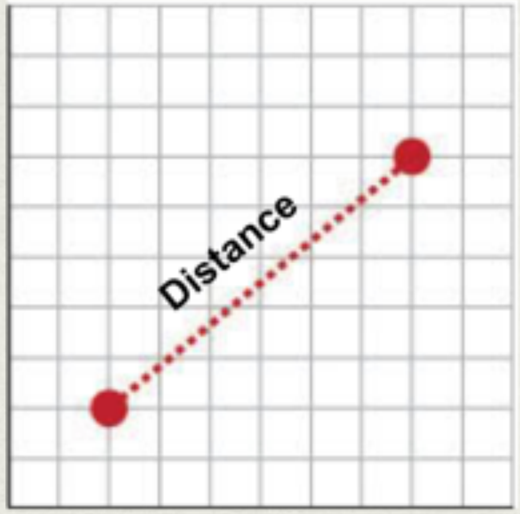
FAMILIES OF RULES

- Unlike voting, JA focused more on impossibility results than on the systematic studies of aggregation rules.
  - *Exceptions*: PBP, CBP, sequential rules, quota-based rules, distance-based rules.
- In *voting* and in *KR*, the idea of minimisation (or maximisation) has been exploited.
- Recently, more *families* of rules have been introduced and studied, e.g. using the criterium of minimisation (Lang *et al.* 2014)



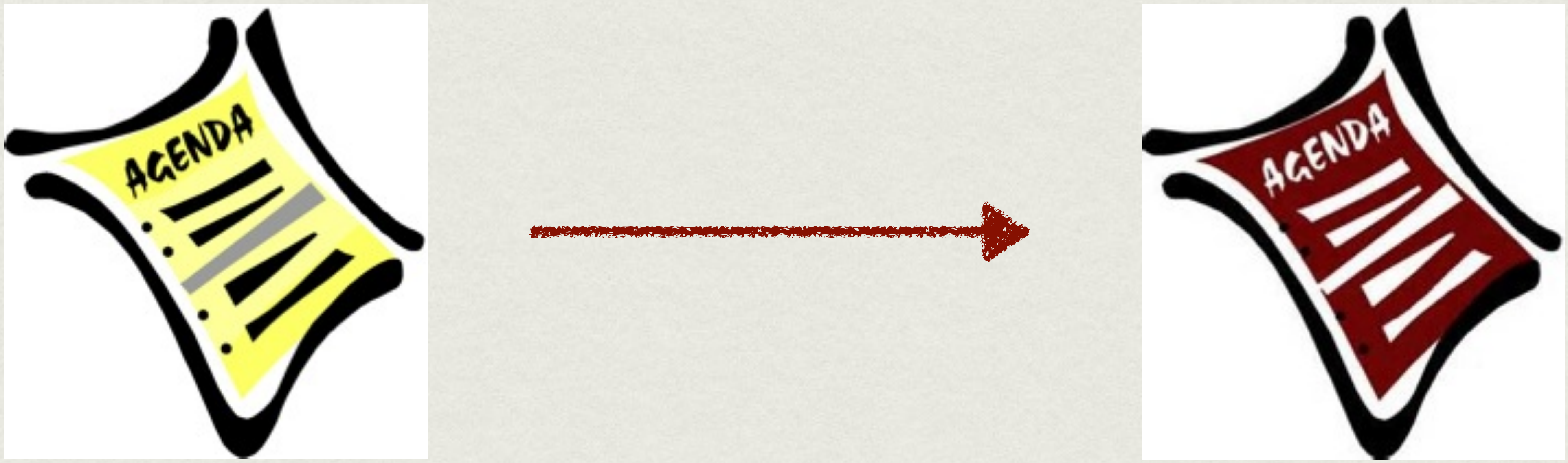
4 FAMILIES OF (NON-DETERMINISTIC) RULES

1. Distance-based rules



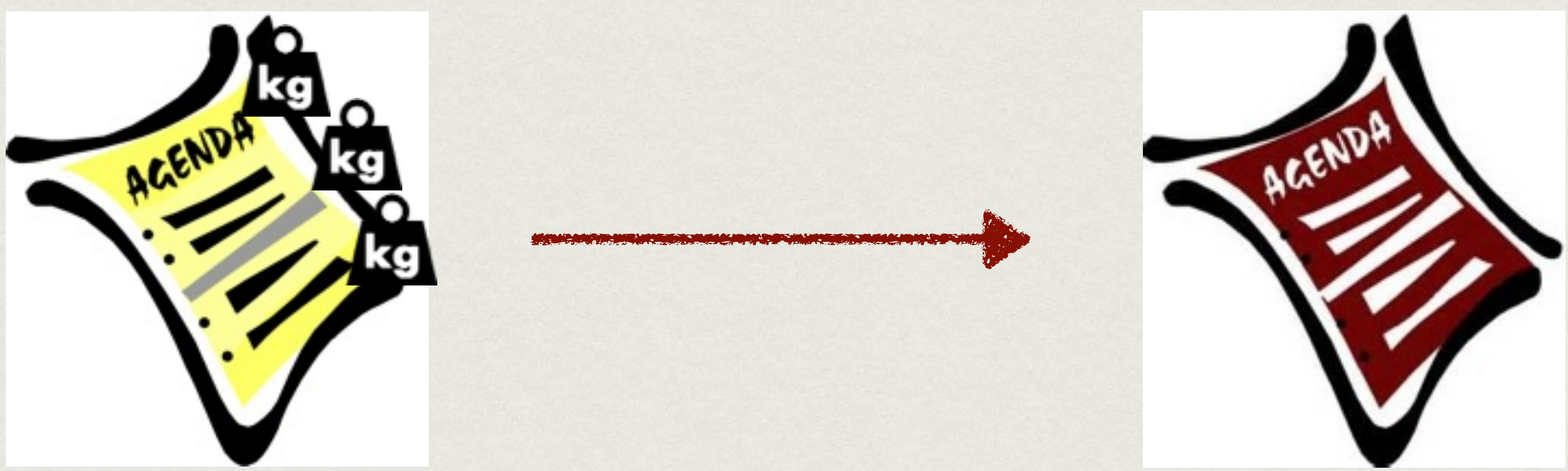
4 FAMILIES OF (NON-DETERMINISTIC) RULES

2. Rules based on the majoritarian judgment set



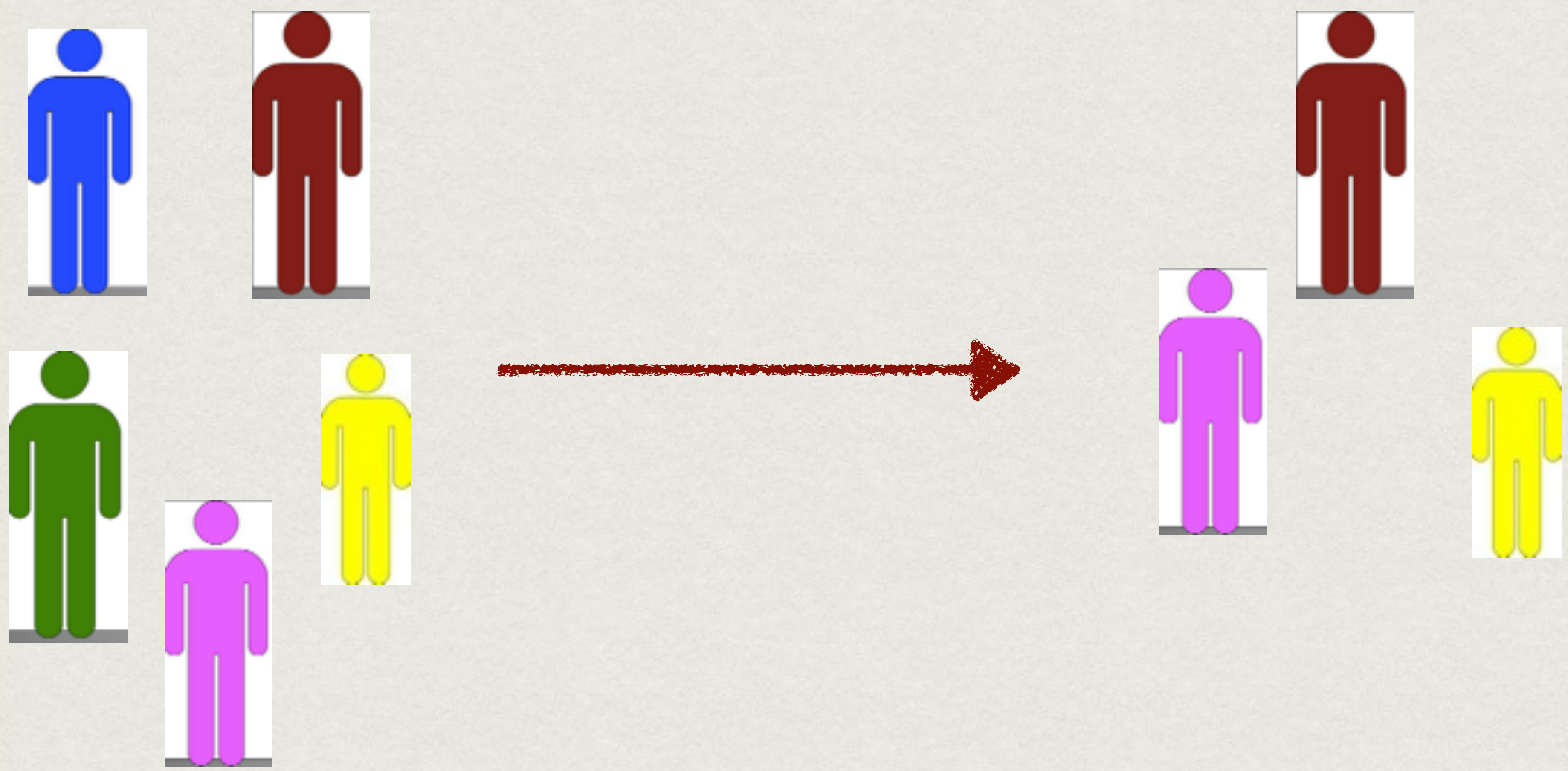
4 FAMILIES OF (NON-DETERMINISTIC) RULES

3. Rules based on the weighted majoritarian judgment set

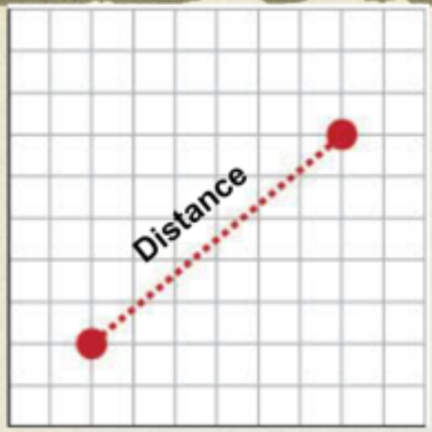


4 FAMILIES OF (NON-DETERMINISTIC) RULES

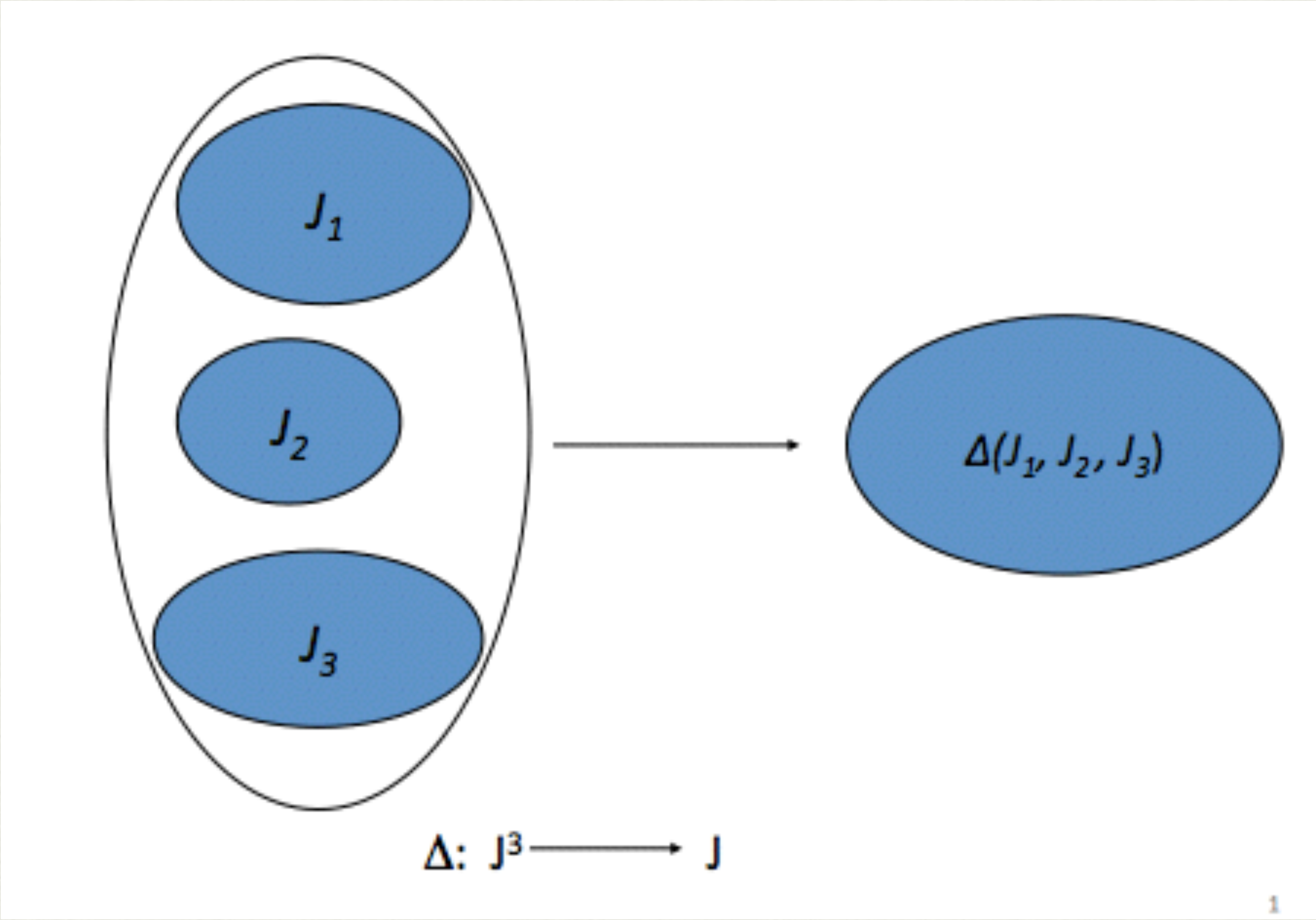
4. Rules based on the removal or change of individual judgments



1. DISTANCE-BASED PROCEDURES (1)



Belief merging (studied in computer science)



1. DISTANCE-BASED PROCEDURES (2)

- Intuition: Identify the judgment set that has minimal distance to the judgments expressed by the individuals.
- Advantage: Aggregation method that ensures a consistent outcome.
- If $(P \wedge Q) \leftrightarrow R$, there are four admissible "judgment sets":

$$J_1 := (1,1,1) \quad , \quad J_2 := (1,0,0)$$
$$J_3 := (0,1,0) \quad , \quad J_4 := (0,0,0)$$

- Define distance between any two judgment sets: e.g. Hamming distance $d(J_1, J_4) = 3$, $d(J_2, J_3) = 2$.
- Integrity constraints avoid that inconsistent outcomes are selected.
- Distance-based procedure captures the idea of reaching a compromise between different individual judgment sets.

1. DISTANCE-BASED PROCEDURES (3)

	$d_H(.,J_1)$	$d_H(.,J_2)$	$d_H(.,J_3)$	$\sum(d_H(.,P))$
$\{p,q,r\}$	0	2	2	4
$\{p,\neg q,\neg r\}$	2	0	2	4
$\{\neg p,q,\neg r\}$	2	2	0	4
$\{\neg p,\neg q,\neg r\}$	3	1	1	5

- *Miller & Osherson (2009)*: 4 general methods for distance-based JA, that do not commit to a specific distance metrics.
- *Duddy & Piggins (2012)* criticised d_H of double counting: $\{p,q,p\wedge q\}$ and $\{p,\neg q,\neg(p\wedge q)\}$ have distance 2 but their disagreement over $(p\wedge q)$ is a consequence of their disagreement over q. Their distance is defined as the smallest number of logically coherent changes needed to convert one judgment set into the other.

NOTATION

- Given a profile *P*, the majoritarian judgment set of P (denoted m(P)) is the output of proposition-wise majority.

- A profile is majority consistent iff *m(P)* is a consistent set of formulas.

	$p \wedge r$	$p \wedge s$	q	$p \wedge q$	t
6 voters	1	1	1	1	1
4 voters	1	1	0	0	1
7 voters	0	0	1	0	0
$m(P)$	1	1	1	0	1

majority inconsistent profile

- A JA rule *f* is majority preserving iff for every majority consistent profile, *f(P)= {m(P)}* (counterpart for JA of Condorcet-consistent voting rules).

2. RULES BASED ON THE MAJORITARIAN JUDGMENT SET (1)

- *Idea*: calculate *m(P)* and, if not consistent, *minimally* remove some issues of the agenda.

Definition (R_{MSA} and R_{MCSA})

- Maximal subagenda rule:
  $R_{MSA}(P) =$ set of all maximal consistent subsets of $m(P)$.
- Maxcard subagenda rule:
  $R_{MCSA}(P) =$ set of all consistent subsets of maximal cardinality of $m(P)$.

2. RULES BASED ON THE MAJORITARIAN JUDGMENT SET (2)

	$p \wedge r$	$p \wedge s$	q	$p \wedge q$	t
6 voters	1	1	1	1	1
4 voters	1	1	0	0	1
7 voters	0	0	1	0	0
	1	1	1		1
$MSA(P)$	1	1		0	1
			1	0	1

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6 voters	1	1	1	1	1
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7 voters	0	0	1	0	0
	1	1	1		1
$MSA(P)$	1	1		0	1
			1	0	1

MCSA(P)

- *MCSA* corresponds to Miller & Oshershon' *Endpoint*$_d$ when d is Hamming.

- Independently, Nehring *et al.* defined *Condorcet admissible set* (corresponding to *MSA*) and *Slater rule* (corresponding to *MCSA*).

3. RULES BASED ON THE WEIGHTED MAJORITARIAN SET (1)

- These rules take into account the support that each agenda item receives from individuals.

- $w(P) = \{\langle\varphi, |P\varphi|\rangle, \varphi \in A\}$ records the support received
- Maxweight rule (*MWA*) outputs all consistent subsets *A'* of the agenda that maximize $w_P(A')$.
- *MWA* is called *Prototype* by Miller & Osherson, *median rule* by Nehring *et al.*, *simple scoring rule* by Dietrich, and can be shown to be *equivalent* to the distance-based rule $F^{dH,Sum}$

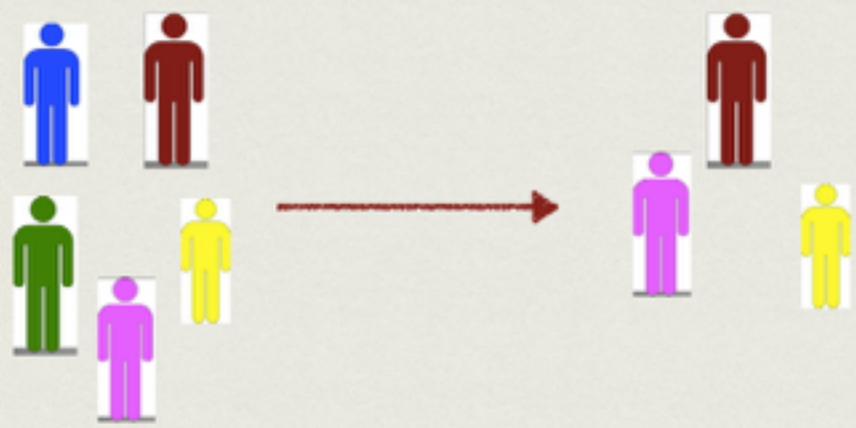
3. RULES BASED ON THE WEIGHTED MAJORITARIAN SET (2)

	$p \wedge r$	$p \wedge s$	q	$p \wedge q$	t
6 voters	Yes	Yes	Yes	Yes	Yes
4 voters	Yes	Yes	No	No	Yes
7 voters	No	No	Yes	No	No
m(P)	Yes	Yes	Yes	No	Yes

$$
\begin{aligned}
w(P) = \{ \quad & \langle p \wedge r, 10\rangle, \langle \neg(p \wedge r), 7\rangle, \\
& \langle p \wedge s, 10\rangle, \langle \neg(p \wedge s), 7\rangle, \\
& \langle q, 13\rangle, \langle \neg q, 4\rangle, \\
& \langle p \wedge q, 6\rangle, \langle \neg(p \wedge q), 11\rangle, \\
& \langle t, 10\rangle, \langle \neg t, 7\rangle\}
\end{aligned}
$$

$$R_{MWA}(P) = \{\{p \wedge r, p \wedge s, q, p \wedge q, t\}\}$$

4. RULES BASED ON THE REMOVAL/CHANGE OF INDIVIDUAL JUDGMENTS (1)



- Instead of minimally changing the agenda, we can *minimally change the profile*.
- *Restriction of P to Q* is $P{\downarrow}Q = \langle J_j \rangle_{j \in Q}$ (sub-profile of P).
- *MSP(P)* is the set of majority-consistent sub-profiles of *P* of maximal length.
- Young JA rule: $Y(P) = \{m(P') \mid P' \in MSP(P)\}$.

4. RULES BASED ON THE REMOVAL/CHANGE OF INDIVIDUAL JUDGMENTS (2)

	$p \wedge r$	$p \wedge s$	q	$p \wedge q$	t
3 ~~6 voters~~	Yes	Yes	Yes	Yes	Yes
4 voters	Yes	Yes	No	No	Yes
7 voters	No	No	Yes	No	No
m(P)	Yes	Yes	Yes	No	Yes

- Removing 3 of the judgment sets $\{p \wedge r, p \wedge s, q, p \wedge q, t\}$ is enough;
- Removing less than 3, or 3 other judgment sets, does not restore majority consistency.

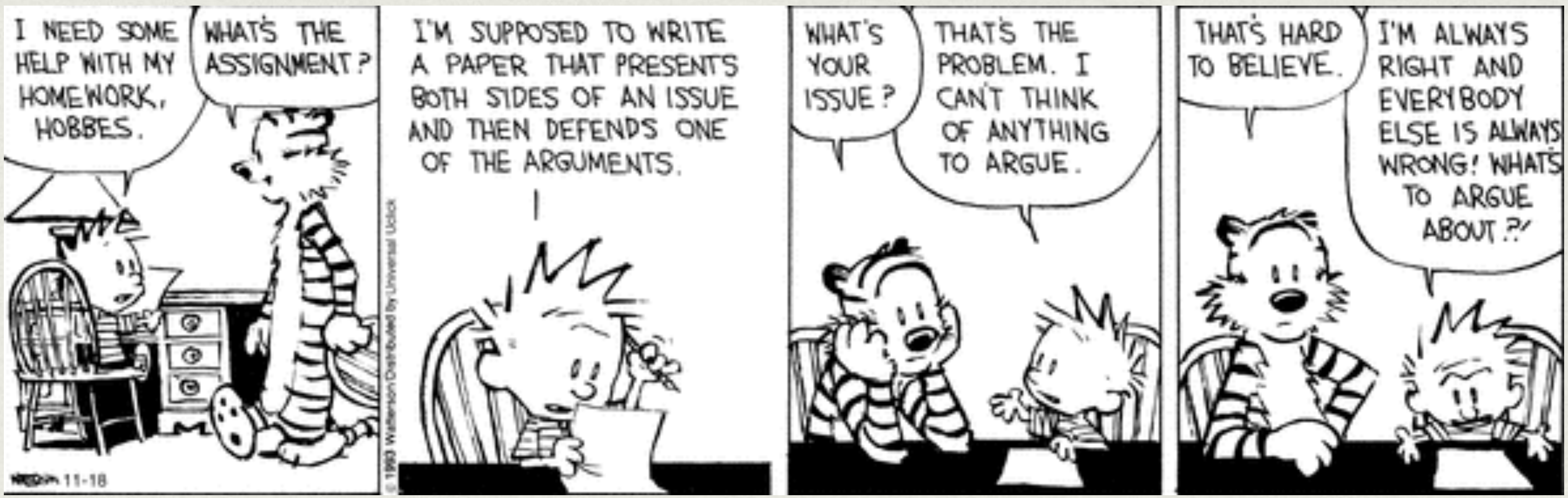
$$R_Y(P) = \{\{\neg(p \wedge r), \neg(p \wedge s), q, \neg(p \wedge q)\}\}$$

SUMMARY OF CORRESPONDENCES

Researchers	Judgment aggregation rules					
Lang, Pigozzi, Slavkovik, van der Torre, Vesic	MSA	MCSA	MWA	RA	MNAC	Young
Nehring, Pivato, Puppe	Condorcet admissible set	Slater Rule	Median Rule			
Miller, Osherson		Endpoint+H.D.	Prototype+H.D.		Full+H.D.	
Dietrich			Simple Scoring Rule			
+ Tr.C.= Voting Rule	Top Cycle	Slater	Kemeny	Ranked Pairs		
+ W.C. = Voting Rule		Copeland		Maximin		Weak Young
Majority Preserving	yes	yes	yes	yes	yes	yes
Complexity of deciding if a judgment is in all collective judgment sets	Π_2^P-c	Θ_2^P-c	Θ_2^P-h	Π_2^P-c	Θ_2^P-c	

Legend:
H.D.= Hamming distance
Tr.C. = Transitivity Constraint
W.C. = Dominance Constraint

ARGUMENTATION



ARGUMENTATION

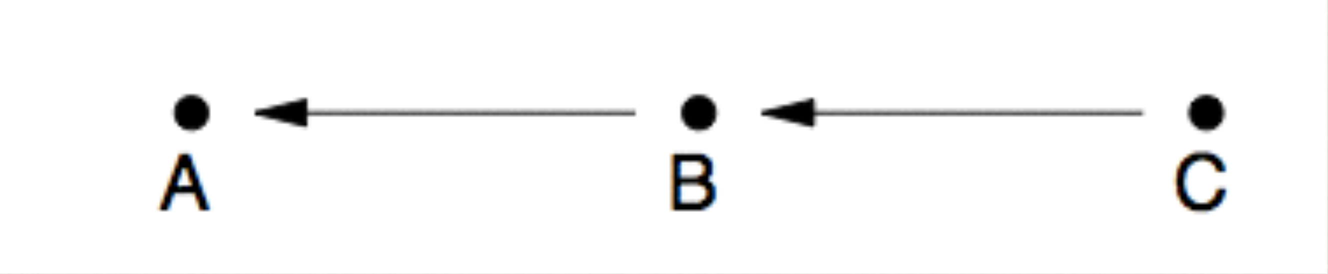
- Hot topic in different disciplines (philosophy, social sciences, computer science...).
- debategraph : a web-based, collaborative idea visualization tool, focusing on complex public policy issues.
- Used by the White House, the UK Foreign and Commonwealth Office, and The Independent newspaper.

ABSTRACT ARGUMENTATION (1)

"Suppose Ralph normally goes fishing on Sundays, but on the Sunday which is Mother's day, he typically visits his parents. Furthermore, in the spring of each leap year his parents take a vacation, so that they cannot be visited."

- Suppose it is Sunday, Mother's day and a leap year.
- Argument A: Ralph goes fishing because it is Sunday.
- Argument B: Ralph does not go fishing because it is Mother's day, so he visits his parents.
- Argument C: Ralph cannot visit his parents, because it is a leap year, so they are on vacation.

ABSTRACT ARGUMENTATION (2)



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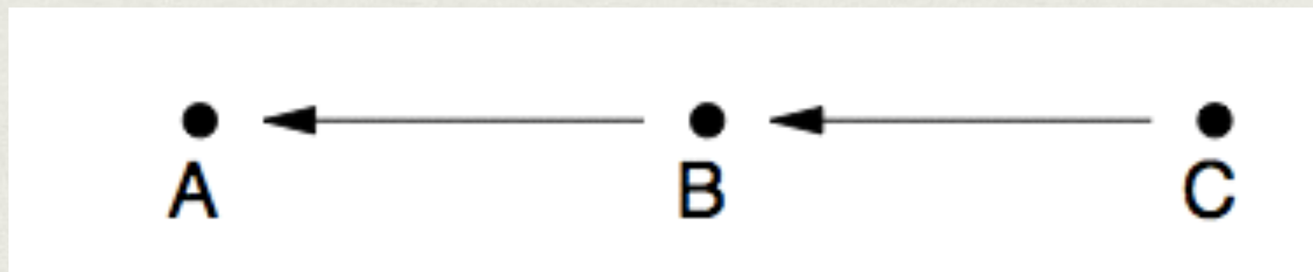


- Su…
- A…
- A…
  M…
- A… t is a
  leap year, so they are on vacation.

Which argument(s) can rationally
be accepted?

ABSTRACT ARGUMENTATION (4)

- Argumentation framework: a set of arguments and a *defeat* relation among them: $AF = (Ar, def)$.

$$C \rightarrow B \rightarrow A$$

- Argumentation theory identifies the sets of arguments (extensions) that can reasonably survive the conflicts expressed in the argumentation framework.
- Each agent assigns a label to each argument:
  - *in* if he accepts the argument
  - *out* if he rejects it
  - *undec* if he abstains from it

- Encoded in argumentation theory is the idea that there exist different rationalities (different semantics).

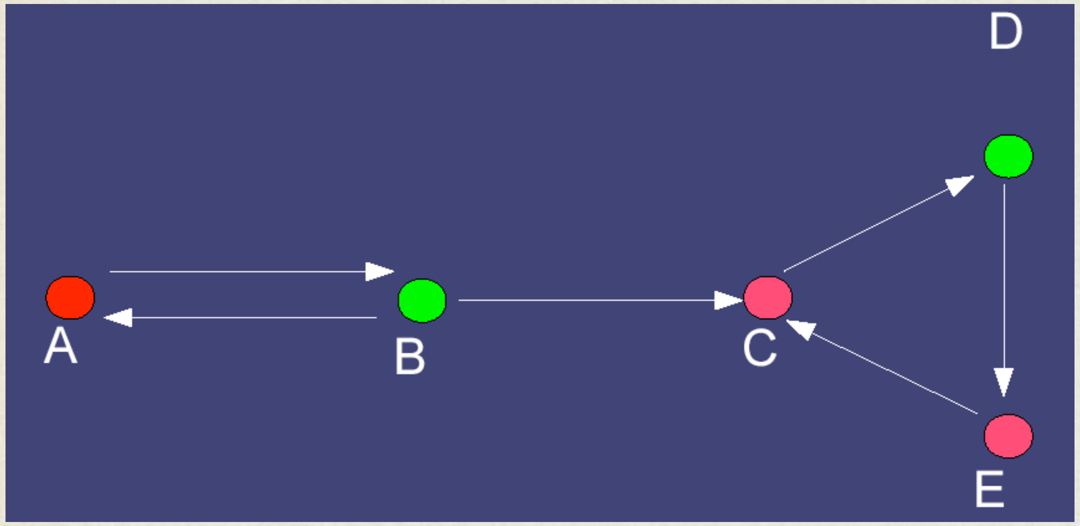
ABSTRACT ARGUMENTATION (5)

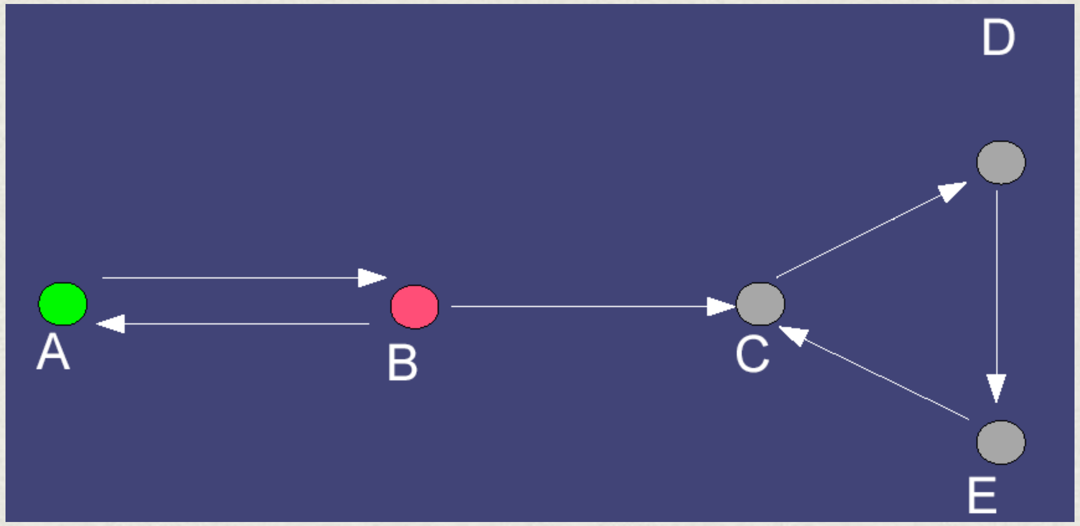
We use the argument labelling approach to define the argument based semantics. A labelling is a total function

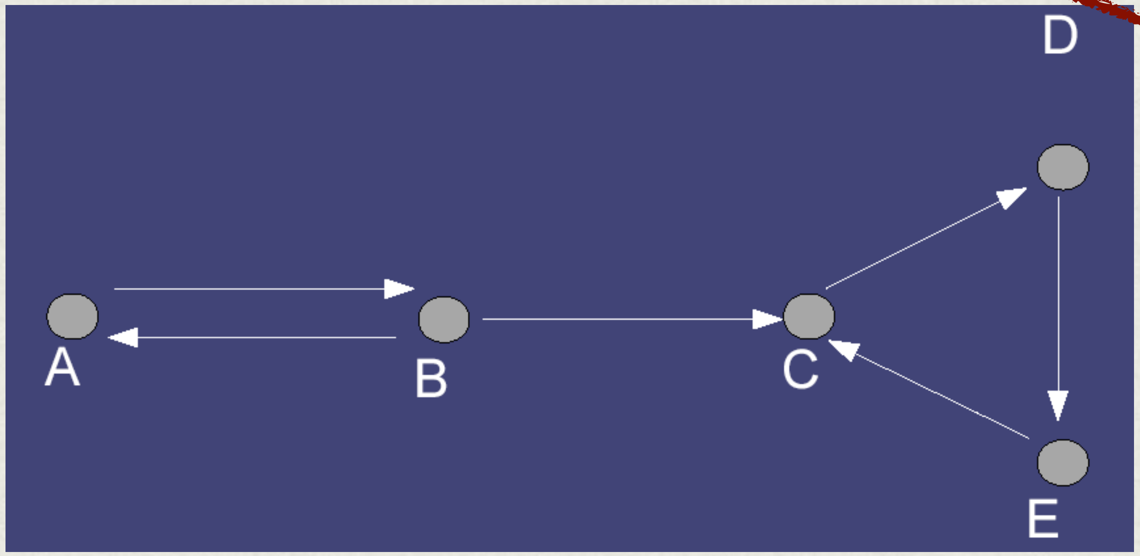
$$\mathscr{L} : Ar \rightarrow \{in, out, undec\}$$

- An argument is (legally) *in* iff all its defeaters are *out*.
- An argument is (legally) *out* iff it has at least a defeater that is labelled *in*. $\Rightarrow$ *Gunfight rules*
- *A* is (legally) *undec* iff it has at least a defeater that is labelled *undec* and has no defeater labelled *in*.

ABSTRACT ARGUMENTATION (6)







Different labellings (= individual positions) possible

ABSTRACT ARGUMENTATION (7)

Definition

Let $\mathscr{L}$ be a labelling of argumentation framework (Ar, def). We say that $\mathscr{L}$ is conflict-free iff for each $A, B \in Ar$, if $\mathscr{L}(A) = in$ and B defeats A, then $\mathscr{L}(B) \neq in$.

Definition

An admissible labelling is a labelling without arguments that are illegally *in* and without arguments that are illegally *out*.

Definition

A complete labelling is a labelling without arguments that are illegally *in*, without arguments that are illegally *out* and without arguments that are illegally *undec*.

JUDGMENT AGGREGATION AS A LABELLING AGGREGATION PROBLEM

F_{AF} is a labellings aggregation operator that assigns a collective labelling $\mathcal{L}_{Coll}$ to each profile $\{\mathcal{L}_1, \ldots, \mathcal{L}_n\}$.

Conditions (UD, CR, anonymity and independence) for F_{AF}:

- **Universal domain**: The domain of F_{AF} is the set of all profiles of individual labellings belonging to semantics $\mathcal{T}_{conflict-free}$, $\mathcal{T}_{admissible}$ or $\mathcal{T}_{complete}$.
- **Collective rationality**: $F_{AF}(\{\mathcal{L}_1, \ldots, \mathcal{L}_n\})$ is a labelling belonging to semantics $\mathcal{T}_{conflict-free}$, $\mathcal{T}_{admissible}$ or $\mathcal{T}_{complete}$.

FROM ONE AGENT TO A GROUP OF AGENTS (1)

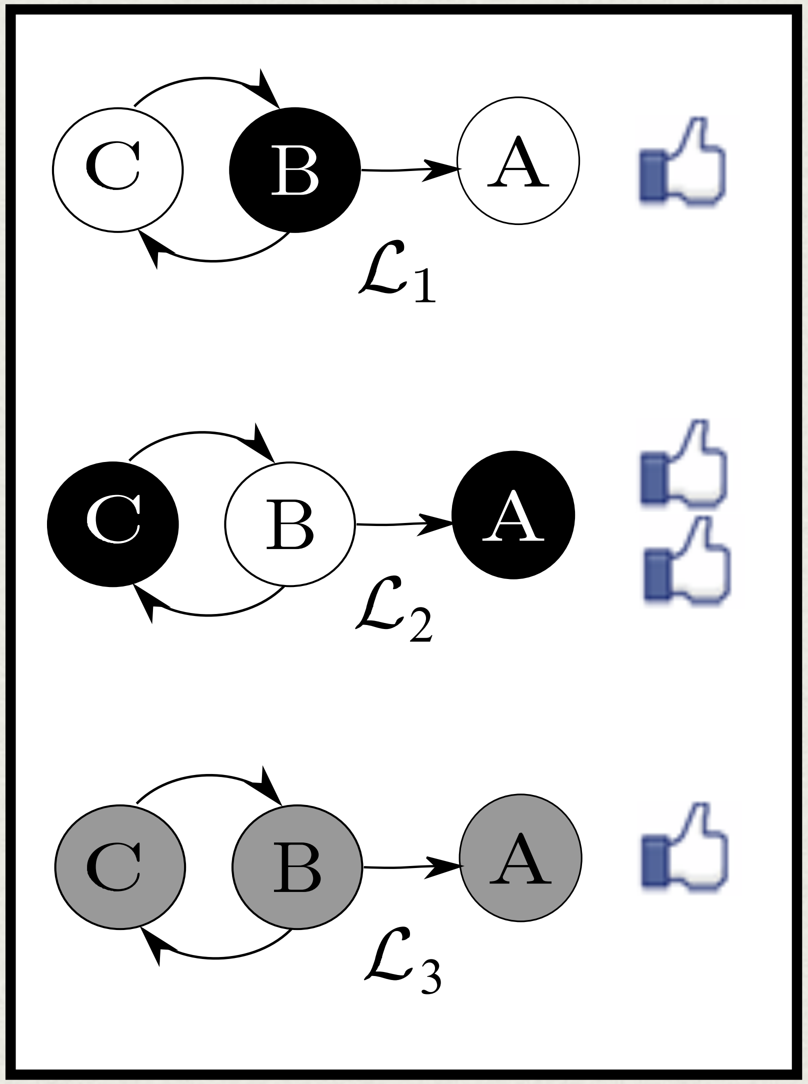
- Abstract argumentation developed in a single agent perspective (or a dialogue).
- A truly MAS (multi-agent) perspective is lacking.
- Few works until now, but growing literature.

FROM ONE AGENT TO A GROUP OF AGENTS (2)

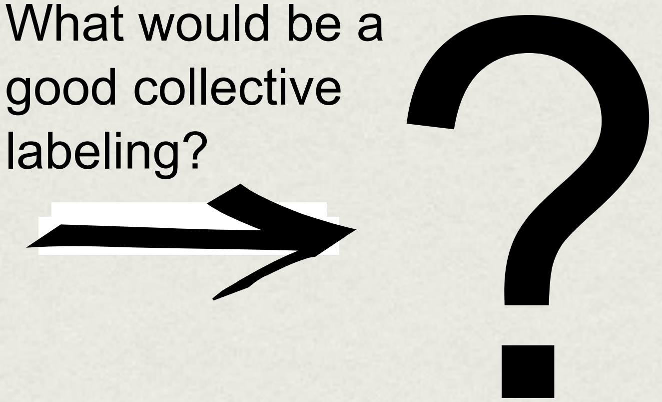
A: The suspect is innocent. Therefore, he should be set free.

B: The suspect was at the crime scene. Therefore, he is not innocent.

C: The suspect was in a bar. Therefore, he is innocent.



What would be a good collective labeling?

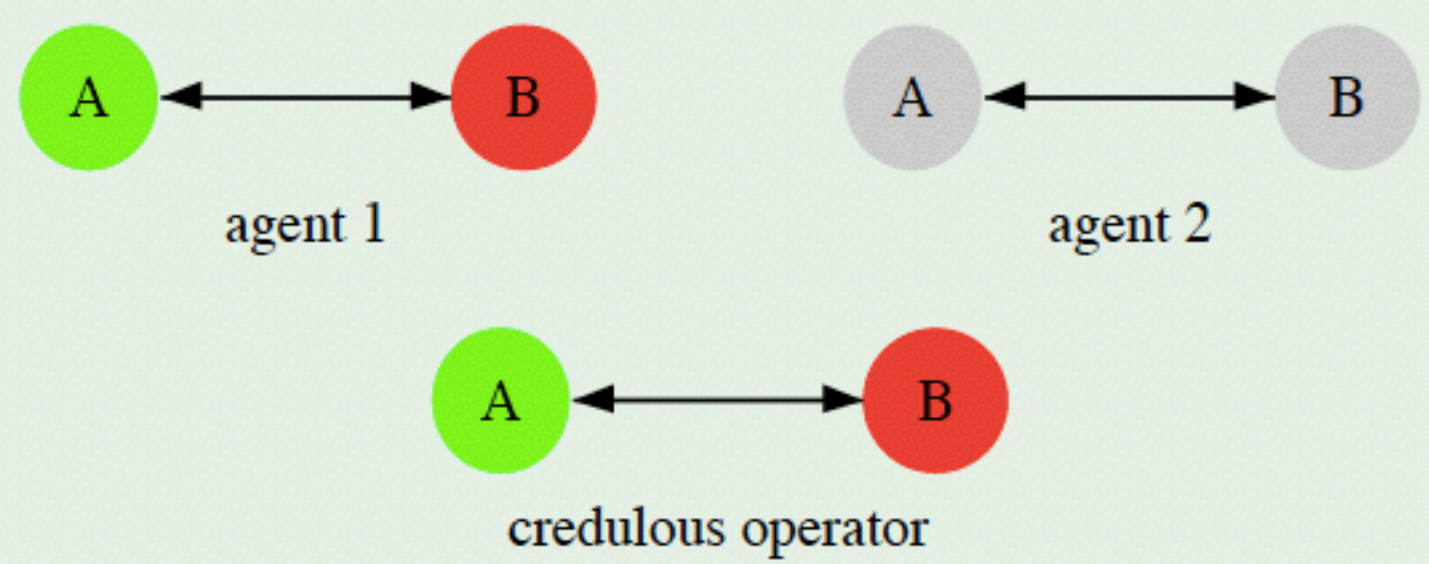


THE CREDULOUS AGGREGATION (1)

First phase: the credulous initial labelling ($\mathcal{L}_{cio}$):

- A is labelled *in* if someone thinks A is *in* and nobody thinks A is *out*.
- A is labelled *out* if someone thinks A is *out* and nobody thinks is *in*.
- A is labelled *undec* in all other cases.

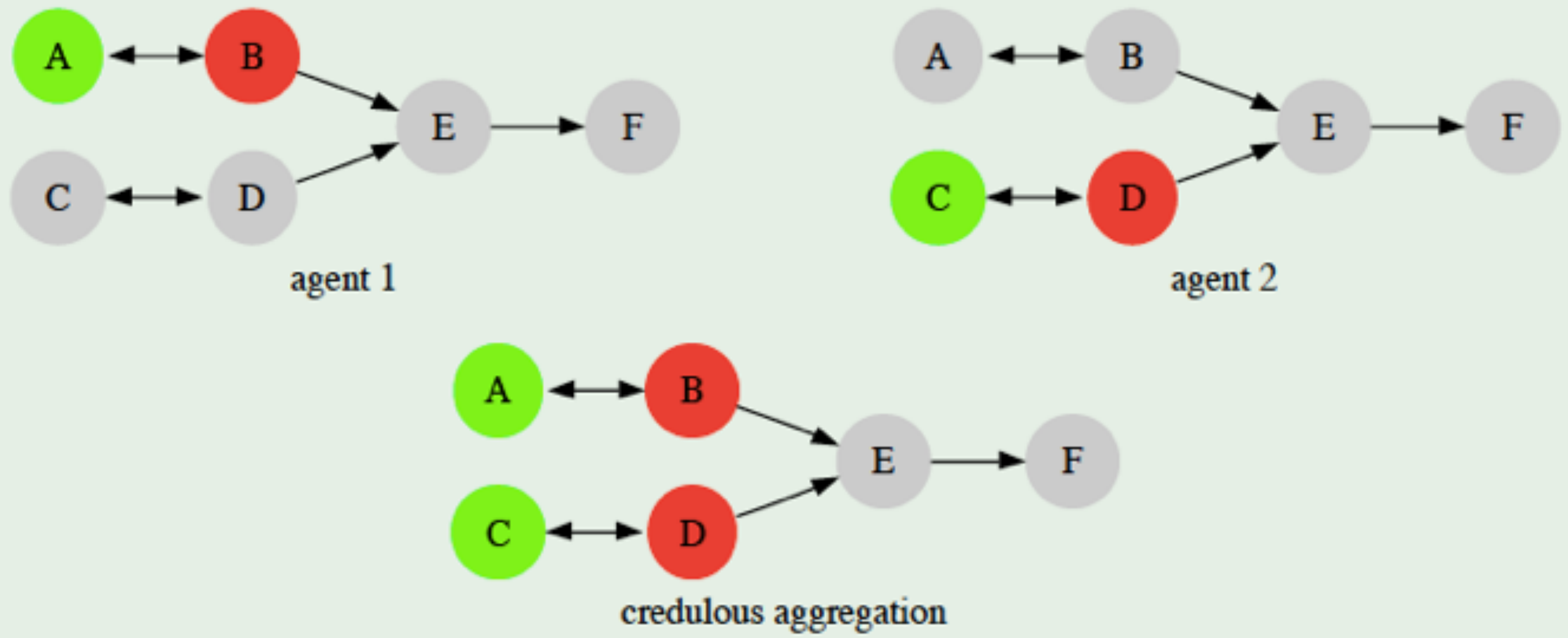
Example



THE CREDULOUS AGGREGATION (2)

$\mathcal{L}_{co}$ satisfies collective rationality under conflict-freeness and admissibility (but **not** under completeness).

Example



$\mathcal{L}_{co}$ can ignore unanimity.

THANK YOU!