System Validation: Hennessy-Milner Logic

Mohammad Mousavi and Jeroen Keiren

### General Overview



### **General Overview**



Complex behaviour of specification



- Complex behaviour of specification
- Concise specification hard to establish



- Complex behaviour of specification
- Concise specification hard to establish
- Why is specification correct?



- Complex behaviour of specification
- Concise specification hard to establish
- Why is specification correct?
- Full behaviour unknown in early stages of development

- Complex behaviour of specification
- Concise specification hard to establish
- Why is specification correct?
- Full behaviour unknown in early stages of development

- Complex behaviour of specification
- Concise specification hard to establish
- Why is specification correct?
- Full behaviour unknown in early stages of development

Solution: express properties outside of behaviour



Fix observable events (interactions with external world)



©Krauss (CC BY-SA 4.0)



Fix observable events (interactions with external world)



©Krauss (CC BY-SA 4.0)

Describe temporal properties using these



Fix observable events (interactions with external world)



©Krauss (CC BY-SA 4.0)

- Describe temporal properties using these
- Verify correctness of properties with respect to some LTS



A scientist interacts with environment

coffee for taking coffee in



A scientist interacts with environment

- coffee for taking coffee in
- coin for producing a coin



A scientist interacts with environment

- coffee for taking coffee in
- coin for producing a coin
- *pub* for producing a publication

A scientist interacts with environment

- coffee for taking coffee in
- coin for producing a coin
- *pub* for producing a publication

• • • •



A scientist interacts with environment

- coffee for taking coffee in
- coin for producing a coin
- *pub* for producing a publication

• • • •



A scientist interacts with environment

- coffee for taking coffee in
- coin for producing a coin
- *pub* for producing a publication

• • • •

Properties of interest

the scientist is not willing to drink coffee now

A scientist interacts with environment

- coffee for taking coffee in
- coin for producing a coin
- *pub* for producing a publication

• • • •

Properties of interest

- the scientist is not willing to drink coffee now
- the scientist is willing to drink both coffee and tea now



A scientist interacts with environment

- coffee for taking coffee in
- coin for producing a coin
- *pub* for producing a publication

• • • •

Properties of interest

- the scientist is not willing to drink coffee now
- the scientist is willing to drink both coffee and tea now
- the scientist will always produce a publication immediately after drinking two coffees in a row

For  $a \in Act$ , Hennessy-Milner formulas  $\varphi, \psi$  are the following:

*true* holds in every state



- *true* holds in every state
- false holds nowhere



true	holds in every state
false	holds nowhere
$\neg \varphi$	holds if $arphi$ does not hold



true	holds in every state
false	holds nowhere
$\neg \varphi$	holds if $arphi$ does not hold
$\varphi \wedge \psi$	holds if both $\varphi$ and $\psi$ hold



true	holds in every state
false	holds nowhere
$\neg \varphi$	holds if $arphi$ does not hold
$\varphi \wedge \psi$	holds if both $\varphi$ and $\psi$ hold
$\varphi \vee \psi$	holds if $arphi$ or $\psi$ holds



holds in every state
holds nowhere
holds if $arphi$ does not hold
holds if both $\varphi$ and $\psi$ hold
holds if $arphi$ or $\psi$ holds
holds if $\neg \varphi \lor \psi$ holds



#### Hennessy-Milner logic Syntax

- true holds in every state
- false holds nowhere
- $\neg \varphi \qquad \qquad \text{holds if } \varphi \text{ does not hold}$
- $\varphi \wedge \psi$  holds if both  $\varphi$  and  $\psi$  hold
- $\varphi \lor \psi \qquad \text{ holds if } \varphi \text{ or } \psi \text{ holds}$
- $\varphi \implies \psi \quad \text{holds if } \neg \varphi \lor \psi \ \text{holds}$
- $\langle a \rangle \varphi \qquad \ \ \, {\rm holds} \ {\rm if} \ {\rm it} \ {\rm is \ possible \ to \ perform \ action \ } a \ {\rm to \ a \ state \ satisfying \ } \varphi$



#### Hennessy-Milner logic Syntax

 $\varphi$ 

- true holds in every state
- false holds nowhere
- $\neg \varphi \qquad \qquad \text{holds if } \varphi \text{ does not hold}$
- $\varphi \wedge \psi$  holds if both  $\varphi$  and  $\psi$  hold

$$\varphi \lor \psi$$
 holds if  $\varphi$  or  $\psi$  holds

$$\implies \psi \quad \text{holds if } \neg \varphi \lor \psi \text{ holds}$$

- $\langle a \rangle \varphi \qquad \ \ \, {\rm holds} \ {\rm if} \ {\rm it} \ {\rm is \ possible \ to \ perform \ action \ } a \ {\rm to \ a \ state \ satisfying \ } \varphi$
- $[a] \varphi$  holds if all successors reached by performing action a satisfy  $\varphi$

#### Hennessy-Milner logic Syntax

 $\langle a \rangle \varphi$ 

 $[a]\varphi$ 

- true holds in every state
- *false* holds nowhere
- $\neg \varphi \qquad \qquad \text{holds if } \varphi \text{ does not hold}$
- $\varphi \wedge \psi$  holds if both  $\varphi$  and  $\psi$  hold
- $\varphi \lor \psi \qquad \text{ holds if } \varphi \text{ or } \psi \text{ holds}$
- $\varphi \implies \psi \quad \text{holds if } \neg \varphi \lor \psi \ \text{holds}$ 
  - holds if it is possible to perform action a to a state satisfying arphi
  - holds if all successors reached by performing action a satisfy arphi



 $\neg \langle coffee \rangle true$ 



 $\neg \langle coffee \rangle true$  or [coffee] false



 $\neg \langle coffee \rangle true$  or [coffee]false

the scientist is willing to drink both coffee and tea now

 $\neg \langle coffee \rangle true$  or [coffee]false

the scientist is willing to drink both coffee and tea now

 $\langle \textit{coffee} \rangle \textit{true} \land \langle \textit{tea} \rangle \textit{true}$ 



Let  $Act = \{a, b\}$ 

► the process is deadlocked



Let  $Act = \{a, b\}$ 

the process is deadlocked

 $[a] \textit{false} \land [b] \textit{false}$ 



Let  $Act = \{a, b\}$ 

► the process is deadlocked

 $[a] false \land [b] false$ 

▶ the process can execute some action

Let  $Act = \{a, b\}$ 

► the process is deadlocked

[a]false  $\land [b]$ false

the process can execute some action

 $\langle a \rangle$ true  $\lor \langle b \rangle$ true



Let  $Act = \{a, b\}$ 

the process is deadlocked

[a]false  $\land [b]$ false

the process can execute some action

 $\langle a \rangle$ true  $\lor \langle b \rangle$ true

► a must happen next



Let  $Act = \{a, b\}$ 

the process is deadlocked

[a]false  $\land [b]$ false

the process can execute some action

 $\langle a \rangle$ true  $\lor \langle b \rangle$ true

► a must happen next

 $\langle a \rangle$ true  $\wedge [b]$ false



- Identify all subformulas
- Label states with subformulas they satisfy, starting from the smallest subformula (*true*)

Is the HML formula  $\langle a \rangle \langle b \rangle true$  satisfied by the labelled transition system (i.e., by its initial state)?



Subformulas

true  $\langle b \rangle$ true  $\langle a \rangle \langle b \rangle$ true

Is the HML formula  $\langle a \rangle \langle b \rangle true$  satisfied by the labelled transition system (i.e., by its initial state)?



Subformulas

Is the HML formula  $\langle a \rangle \langle b \rangle true$  satisfied by the labelled transition system (i.e., by its initial state)?



Subformulas

 $\langle b \rangle true \langle a \rangle \langle b \rangle true$ 

true

Is the HML formula  $\langle a \rangle \langle b \rangle true$  satisfied by the labelled transition system (i.e., by its initial state)?



Subformulas

true  $\langle b \rangle$ true  $\langle a \rangle \langle b \rangle$ true

Is the HML formula  $[a]\langle b \rangle$  true satisfied?





Is the HML formula  $[a]\langle b \rangle$  true satisfied?





Is the HML formula  $[a]\langle b \rangle$  true satisfied?



Assume  $Act = \{coffee, pub\}$ 

the scientist will produce a publication immediately after having drunk two coffees in a row



Assume  $Act = \{coffee, pub\}$ 

the scientist will produce a publication immediately after having drunk two coffees in a row

 $[coffee][coffee](\langle pub \rangle true \land [coffee]false)$ 



Assume  $Act = \{coffee, pub\}$ 

the scientist will produce a publication immediately after having drunk two coffees in a row

#### $[coffee][coffee](\langle pub \rangle true \land [coffee]false)$

the scientist will always produce a publication immediately after having drunk two coffees in a row

Assume  $Act = \{coffee, pub\}$ 

the scientist will produce a publication immediately after having drunk two coffees in a row

#### $[coffee][coffee](\langle pub \rangle true \land [coffee]false)$

the scientist will always produce a publication immediately after having drunk two coffees in a row not expressible in HML

Assume  $Act = \{coffee, pub\}$ 

the scientist will produce a publication immediately after having drunk two coffees in a row

#### $[coffee][coffee](\langle pub \rangle true \land [coffee]false)$

the scientist will always produce a publication immediately after having drunk two coffees in a row not expressible in HML

#### Observations

There are relevant properties that cannot be expressed in HML. HML is restricted to a finite depth.

- Behavioural equivalences not always suitable for verification
- Hennessy-Milner logic provides alternative way to describe properties
- Only properties of finite depth can be described

### General Overview



# Thank you very much.

