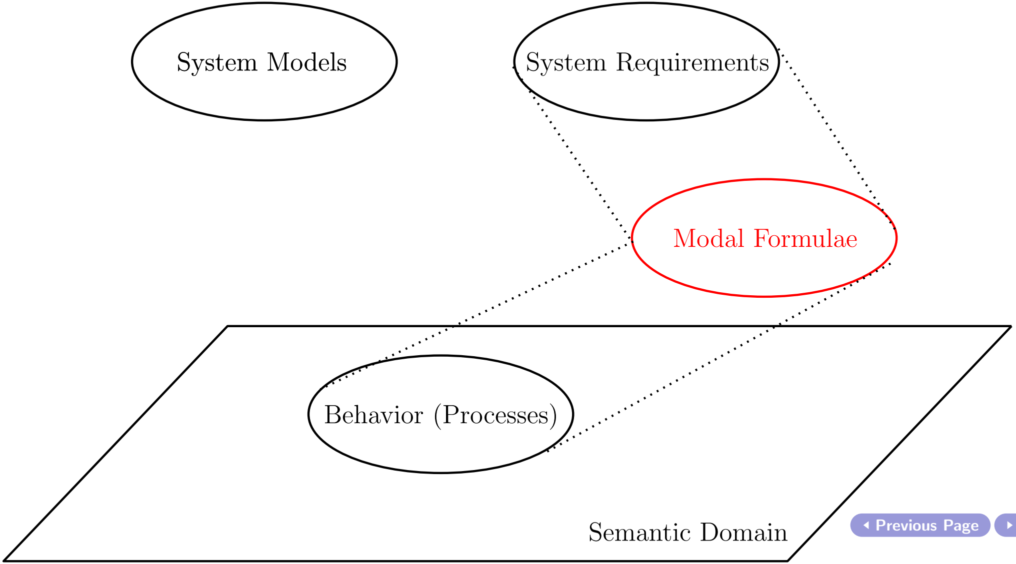
System Validation: Extensions of Hennessy-Milner Logic

Mohammad Mousavi and Jeroen Keiren

General Overview



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  - all reachable states satisfy φ

$$Inv(\varphi) = \varphi \wedge [true]\varphi \wedge [true][true]\varphi \wedge \cdots$$

  - there is a reachable state which satisfies φ

$$Pos(\varphi) = \varphi \vee \langle true\rangle\varphi \vee \langle true\rangle\langle true\rangle\varphi \vee \cdots$$

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- $[A]\varphi$ denotes $[a_1]\varphi \wedge \cdots \wedge [a_n]\varphi$ and $[\emptyset]\varphi = \mathit{true}$

Action formula

A described using the following syntax ($a \in Act$):

$$A, B ::= \mathit{false} \mid \mathit{true} \mid a \mid \overline{A} \mid A \cup B \mid A \cap B$$

where $\overline{A} = Act \setminus A$, *true* matches all actions, *false* matches no action.

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Limitations of HML revisited

Formulas for properties that cannot be expressed in HML

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Why not use recursion?

- $\mathit{Inev}(\varphi)$ expressed by $X \stackrel{\text{def}}{=} \varphi \vee [\mathit{true}]X$
- $\mathit{Safe}(\varphi)$ expressed by $X \stackrel{\text{def}}{=} \varphi \wedge \langle \mathit{true} \rangle X$

Summary

- Allowing sets inside modalities $\implies$ more compact formulas
- Regular HML allows describing properties of infinite depth
- Some desirable properties cannot be described using regular HML

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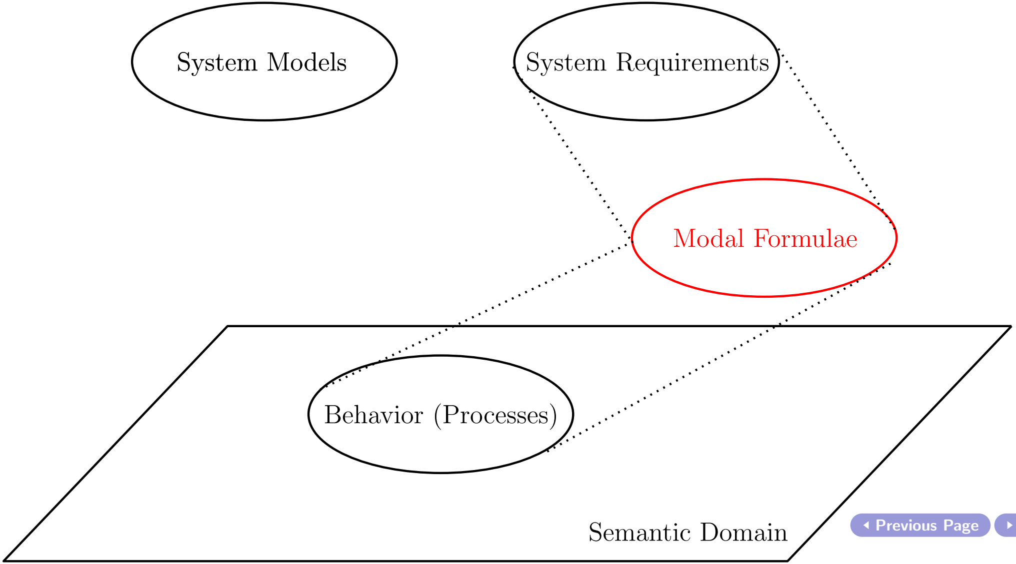

Thank you very much.