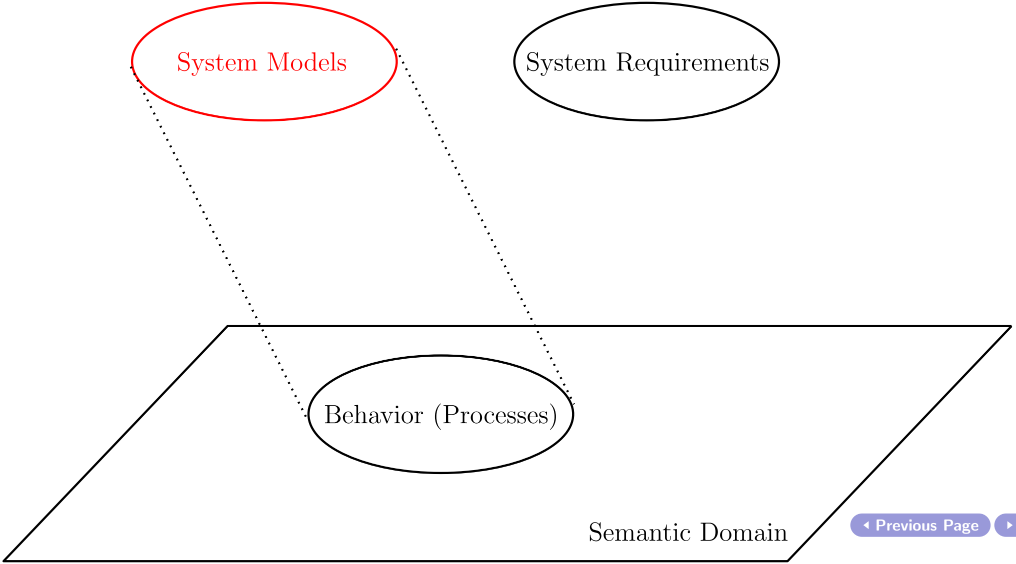
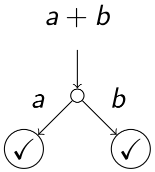
System Validation: Describing Sequential Processes

Mohammad Mousavi and Jeroen Keiren

General Overview

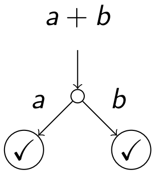


Alternative composition



Alternative composition

- Syntax $p + q$
- Intuition the process behaves as either p or q



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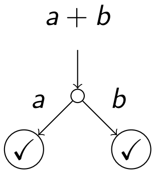

Axioms

A1	$x + y = y + x$
A2	$x + (y + z) = (x + y) + z$
A3	$x + x = x$

Write $x \subseteq y$ for $x + y = y$.

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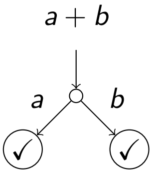

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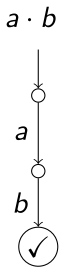
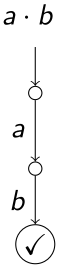
Sequential composition



Sequential composition

- Syntax $p \cdot q$
- Intuition the process behaves as p and upon termination of p, as q.



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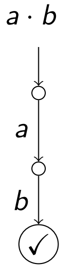
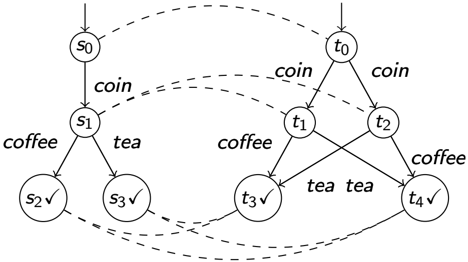


- Syntax $p \cdot q$
- Intuition the process behaves as p and upon termination of p, as q.

Axioms

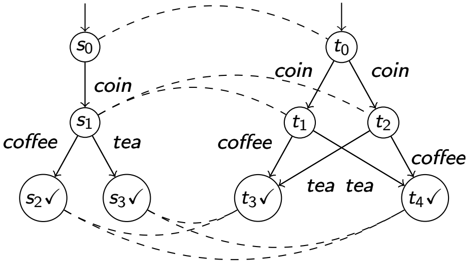
$$
\begin{array}{ll}
\text{A4} & (x + y) \cdot z = x \cdot z + y \cdot z \\
\text{A5} & (x \cdot y) \cdot z = x \cdot (y \cdot z)
\end{array}
$$

Example



s_0
t_0
coin
coin
coin
s_1
t_1
t_2
coffee
tea
coffee
coffee
s_2 ✓
s_3 ✓
t_3 ✓
tea tea
t_4 ✓

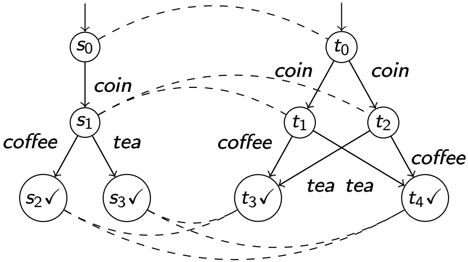
Example



$$coin \cdot (coffee + tea)$$

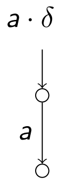
$$(coin \cdot (coffee + tea)) + (coin \cdot (tea + coffee))$$

Example



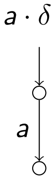
$$\begin{aligned} & coin \cdot (coffee + tea) \\ \overset{A1,A3}{=} \; & (coin \cdot (coffee + tea)) + (coin \cdot (tea + coffee)) \end{aligned}$$

Deadlock



Deadlock

- Syntax δ
- Intuition a process that cannot do anything but let time pass



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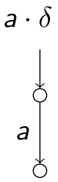

Axioms

$$\boxed{\begin{array}{ll} \text{A6} & x + \delta = x \\ \text{A7} & \delta \cdot x = \delta \end{array}}$$

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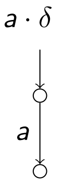

Axioms

A6	$x + \delta = x$
A7	$\delta \cdot x = \delta$

Conditional operator

$$(n < 42) \rightarrow a \diamond b$$

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Axioms

Cond1	$true \to x \diamond y = x$
Cond2	$false \to x \diamond y = y$
THEN	$c \to x = c \to x \diamond \delta$

Conditional operator

$$(n < 42) \to a \diamond b$$

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Cond1	$true \to x \diamond y = x$
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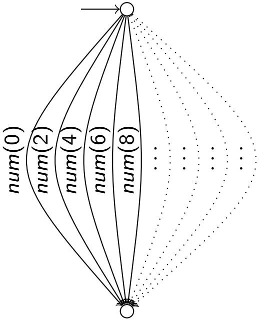
Sum operator

$\sum_{v:Nat} num(2 * v);$



Sum operator

- Syntax $\sum_{d:D} X(d)$
- Intuition generalize alternative composition: may behave as $X(d)$, for each value d of type D

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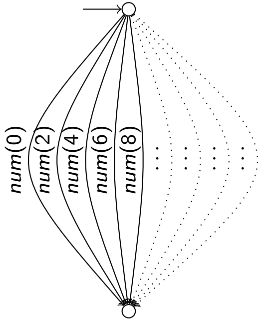

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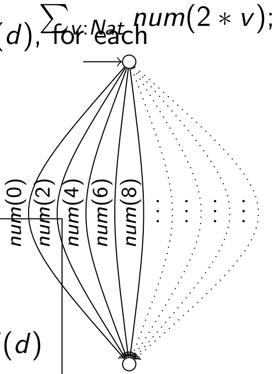

Axioms

SUM1	$\sum_{d:D} x = x$
SUM3	$\sum_{d:D} X(d) = X(e) + \sum_{d:D} X(d)$
SUM4	$\sum_{d:D}(X(d) + Y(d)) = \sum_{d:D} X(d) + \sum_{d:D} Y(d)$
SUM5	$(\sum_{d:D} X(d))\cdot y = \sum_{d:D} X(d)\cdot y$

Example

One time usable buffer, with messages of type Message

$$\sum_{m:Message} read(m) \cdot forward(m)$$

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Problem How to handle repetition?

Recursion

Define set of equations with variables as left hand side:

$$P = x$$

where x a process, that can refer to variables such as P

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Define set of equations with variables as left hand side:

$$P = x$$

where x a process, that can refer to variables such as P

- allows definition of infinite processes
- can store data in parameters

Example

Reusable 1-place FIFO buffer, with messages of type Message

$$Buffer = \sum_{m:Message} read(m) \cdot forward(m) \cdot Buffer$$

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$$Buffer = \sum_{m:Message} read(m) \cdot forward(m) \cdot Buffer$$

or, in mCRL2:

```
sort Message;

act read,forward: Message;

proc Buffer = sum m: Message . read(m) . forward(m) . Buffer;

init Buffer;
```

Example

Infinite queue

$$Queue(l{:}List(Message)) = \sum_{m:Message} read(m) \cdot Queue(l \lhd m)$$
$$+ (l \neq [] \rightarrow forward(head(l)) \cdot Queue(tail(l)))$$

Example

Infinite queue

$$\begin{aligned} \mathit{Queue}(l{:}\mathit{List}(\mathit{Message})) = & \sum_{m:\mathit{Message}} \mathit{read}(m) \cdot \mathit{Queue}(l \triangleleft m) \\ & + (l \neq [] \to \mathit{forward}(\mathit{head}(l)) \cdot \mathit{Queue}(\mathit{tail}(l))) \end{aligned}$$

Example

Infinite queue

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or, in mCRL2:

```
sort Message;

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proc Queue(l: List(Message)) =
       sum m: Message . read(m) . Queue(l <| m)
   + (l != []) -> forward(head(l)) . Queue(tail(l));

init Queue([]);
```

Thank you very much.