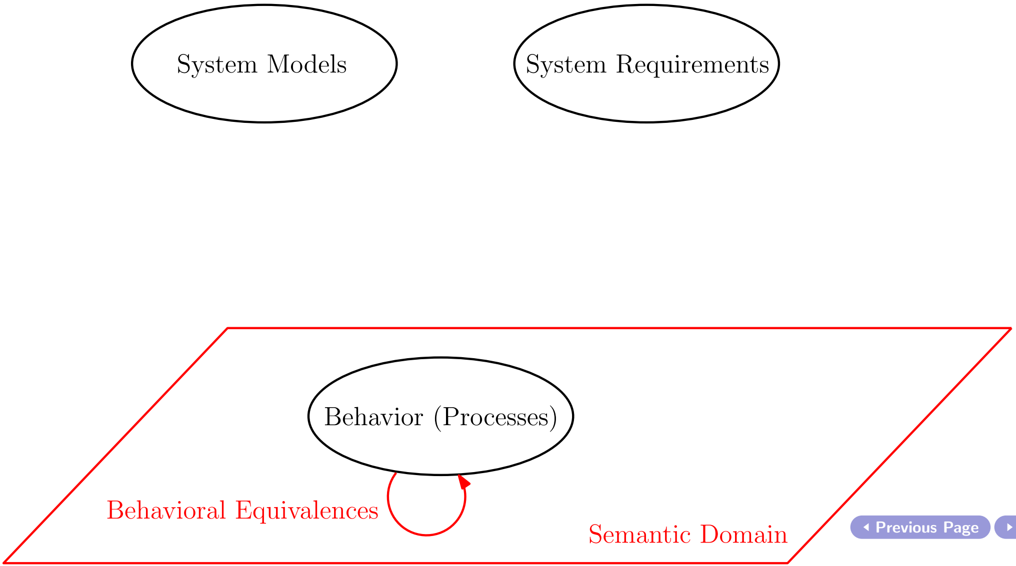
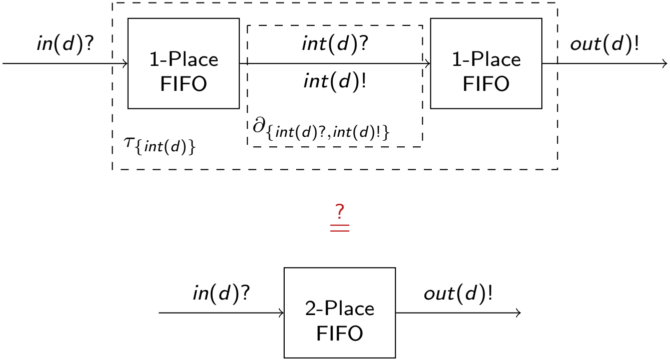
System Validation: Weak Behavioral Equivalences

Mohammad Mousavi and Jeroen Keiren

General Overview



Motivation
Verifying two-place buffer



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- Internal actions should be invisible to the outside world

Weak Equivalences
Idea

- Internal actions should be invisible to the outside world
- τ: The collective name for all invisible actions

Weak Equivalences
Idea

- Internal actions should be invisible to the outside world
- τ: The collective name for all invisible actions
- Adapt behavioral equivalence to neglect τ

Trace Equivalence

Traces of a State

For state $t \in S$, $Traces(t)$ is the minimal set satisfying:

1. $\epsilon \in Traces(t)$
2. $\surd \in Traces(t)$ when $t \in T$
3. $a\sigma \in Traces(t)$ when $t \xrightarrow{a} t'$, and $\sigma \in Traces(t')$

Trace Equivalence

For states t, t', t is trace equivalent to t' iff $Traces(t) = Traces(t')$.

Weak Trace Equivalence

Weak Traces of a State

For state $t \in S$, $WTraces(t)$ is the minimal set satisfying:

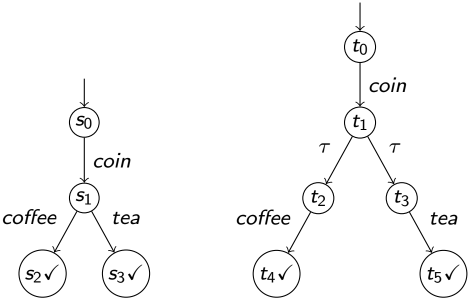
1. $\epsilon \in WTraces(t)$
2. $\surd \in WTraces(t)$ when $t \in T$
3. $a\sigma \in WTraces(t)$ when $t \xrightarrow{a} t'$, $(a \neq \tau)$ and $\sigma \in WTraces(t')$
4. $\sigma \in WTraces(t)$ when $t \xrightarrow{\tau} t'$ and $\sigma \in WTraces(t')$

Weak Trace Equivalence

For states t, t', t is trace equivalent to t' iff
$WTraces(t) = WTraces(t') Traces(t) = Traces(t')$.

Weak Traces

Example

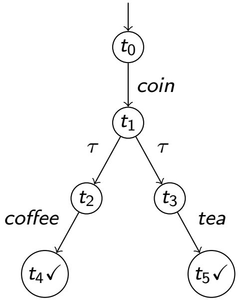


Weak Traces

Example

1. $\epsilon \in WTraces(t)$,
2. $\surd \in WTraces(t)$ when $t \in T$,
3. $a\sigma \in WTraces(t)$ when $t \xrightarrow{a} t'$ and $\sigma \in WTraces(t')$,
4. $\sigma \in WTraces(t)$ when $t \xrightarrow{\tau} t'$ and $\sigma \in WTraces(t')$.

What are $WTraces(s_0)$ and $WTraces(t_0)$?



Weak Traces

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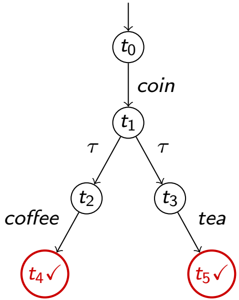

- $WTraces(t_4) = WTraces(t_5) = \{\epsilon, \surd\}$,

Weak Traces
Example

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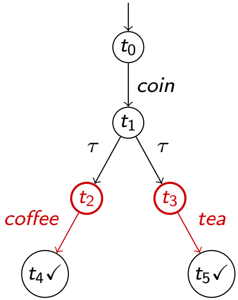

- $WTraces(t_4) = WTraces(t_5) = \{\epsilon, \surd\}$,
- $WTraces(t_2) = \{\epsilon, coffee, coffee\surd\}$, $WTraces(t_3) = \{\epsilon, tea, tea\surd\}$,

Weak Traces
Example

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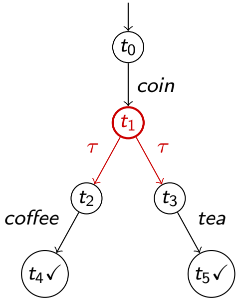
What are $WTraces(s_0)$ and $WTraces(t_0)$?



- $WTraces(t_4) = WTraces(t_5) = \{\epsilon, \surd\}$,
- $WTraces(t_2) = \{\epsilon, coffee, coffee\surd\}$, $WTraces(t_3) = \{\epsilon, tea, tea\surd\}$,
- $WTraces(t_1) = \{\epsilon, coffee, tea, coffee\surd, tea\surd\}$,

Weak Traces
Example

1. $\epsilon \in WTraces(t)$,
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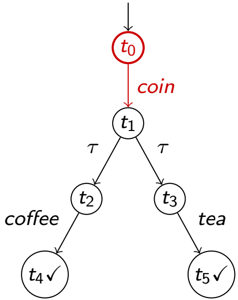
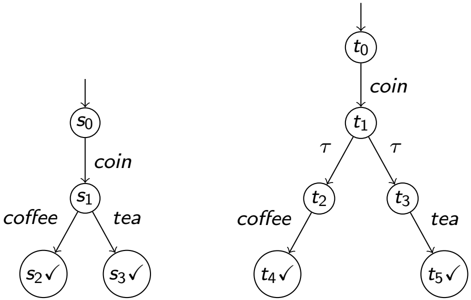
What are $WTraces(s_0)$ and $WTraces(t_0)$?



- $WTraces(t_1) = \{\epsilon, coffee, tea, coffee\surd, tea\surd\}$,
- $WTraces(t_0) = \{\epsilon, coin, coin\ coffee, coin\ tea, coin\ coffee\surd, coin\ tea\surd\}$.

Weak Trace Equivalence

Observation



$WTraces(s_0) = WTraces(t_0) = \{\ \epsilon, coin, coin\ coffee, coin\ tea,$
$coin\ coffee\surd, coin\ tea\surd\}$

Moral of the Story: Weak Trace equivalence is too coarse

Weak Bisimulations

Idea

1. Mimic a-transition by same transition possibly with (stuttering) τ-transitions before and/or after
2. τ-transition can be mimicked by remaining in same state (making no transition)

Weak Bisimulation

Strong Bisimulation

$R \subseteq S \times S$ is strong bisimulation iff
for $s, t \in S$ s.t. $s \; R \; t$, and $a \in Act$:

- if $s \xrightarrow{a} s'$ then
  - $\exists_{t' \in S}$ s.t. $t \xrightarrow{a} t'$ and $s' \; R \; t'$,
- if $s \in T$ then $t \in T$.

and vice versa.

Weak Bisimulation

Weak Bisimulation

$R \subseteq S \times S$ is weak bisimulation iff
for $s, t \in S$ s.t. $s\ R\ t$, and $a \in Act$:

- if $s \xrightarrow{a} s'$ then
  - $a = \tau$ and $s'\ R\ t$, or
  - $\exists_{t_1', t_2', t' \in S}$ s.t. $t \xrightarrow{\tau}{}^{*} t_1' \xrightarrow{a} t_2' \xrightarrow{\tau}{}^{*} t'$ and $s'\ R\ t'$,
- if $s \in T$ then $\exists_{t' \in S}\, t \xrightarrow{\tau}{}^{*} t'$ and $t' \in T$.

and vice versa.

Strong Bisimulation

$R \subseteq S \times S$ is strong bisimulation iff
for $s, t \in S$ s.t. $s\ R\ t$, and $a \in Act$:

- if $s \xrightarrow{a} s'$ then
  - $\exists_{t' \in S}$ s.t. $t \xrightarrow{a} t'$ and $s'\ R\ t'$,
- if $s \in T$ then $t \in T$.

and vice versa.

Branching Bisimulation

Branching Bisimulation

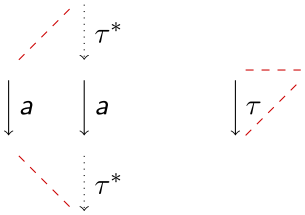
$R \subseteq S \times S$ is branching bisimulation iff
for $s, t \in S$ s.t. $s\ R\ t$, and $a \in Act$:

- if $s \xrightarrow{a} s'$ then
  - $a = \tau$ and $s'\ R\ t$, or
  - $\exists_{t'_1, t' \in S}$ s.t. $t \xrightarrow{\tau}{}^{*} t'_1 \xrightarrow{a} t'$, $s\ R\ t'_1$ and $s'\ R\ t'$,
- if $s \in T$ then $\exists_{t' \in S} t \xrightarrow{\tau}{}^{*} t'$ and $t' \in T$.

and vice versa.

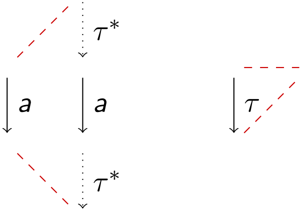
Weak vs. Branching Bisimulation

Weak Bisimulation

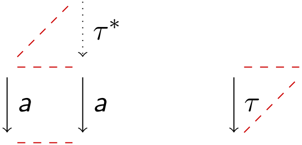


Weak vs. Branching Bisimulation

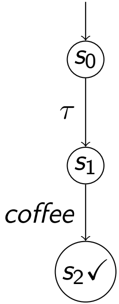
Weak Bisimulation



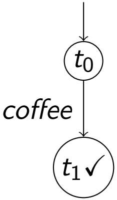
Branching Bisimulation



Weak Bisimulations and Choice



$\underline{\leftrightarrow}_{w,b}$



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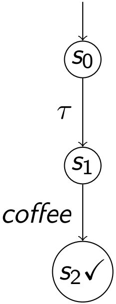

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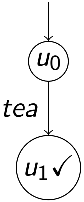

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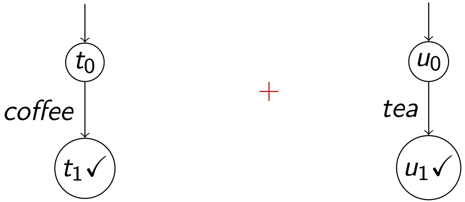

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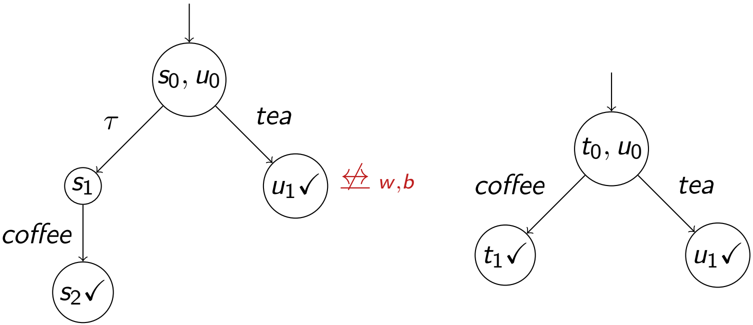

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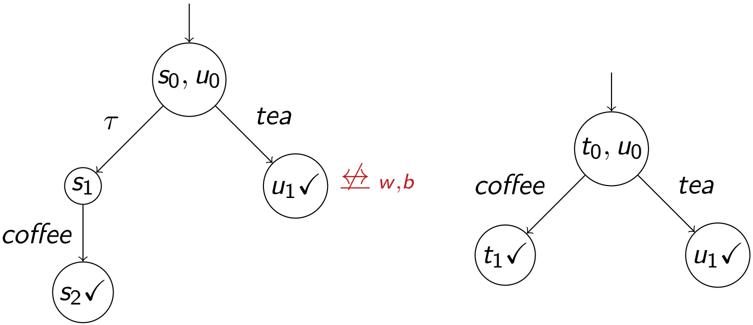


Observation

Weak- and branching bisimulation are not preserved under choice

Root Condition

Basic Idea

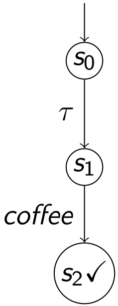
For a branching (or weak) bisimulation to be a congruence with respect to choice, the first τ-transition should be mimicked by a τ transition.

Rootedness

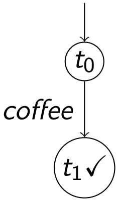
Two state s, t are rooted branching bisimilar if

- there exists a branching bisimulation relation R such that $s\ R\ t$ and
- if $s \xrightarrow{a} s'$ then there is $t' \in S$ s.t. $t \xrightarrow{a} t'$ and $s' \underline{\leftrightarrow}_b t'$, and
- if $t \xrightarrow{a} t'$ then there is $s' \in S$ s.t. $s \xrightarrow{a} s'$ and $s' \underline{\leftrightarrow}_b t'$, and

Weak Bisimulations and Choice



$\underline{\leftrightarrow}_{rw,rb}$ (?)



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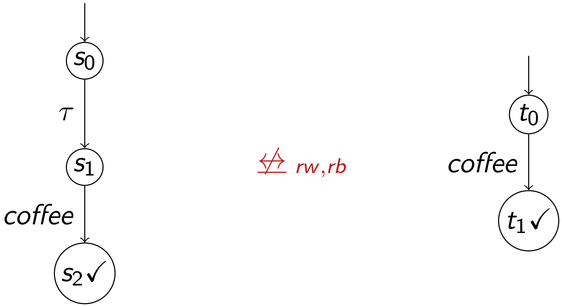
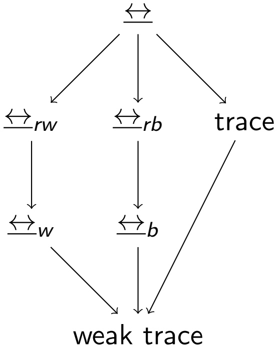


Van Glabbeek's Spectrum

The Treated Part



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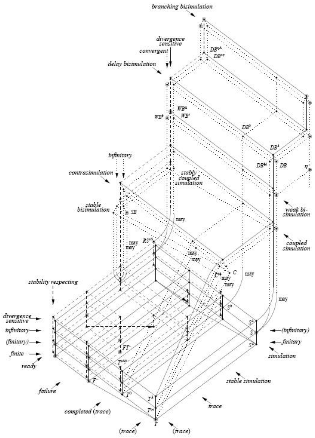
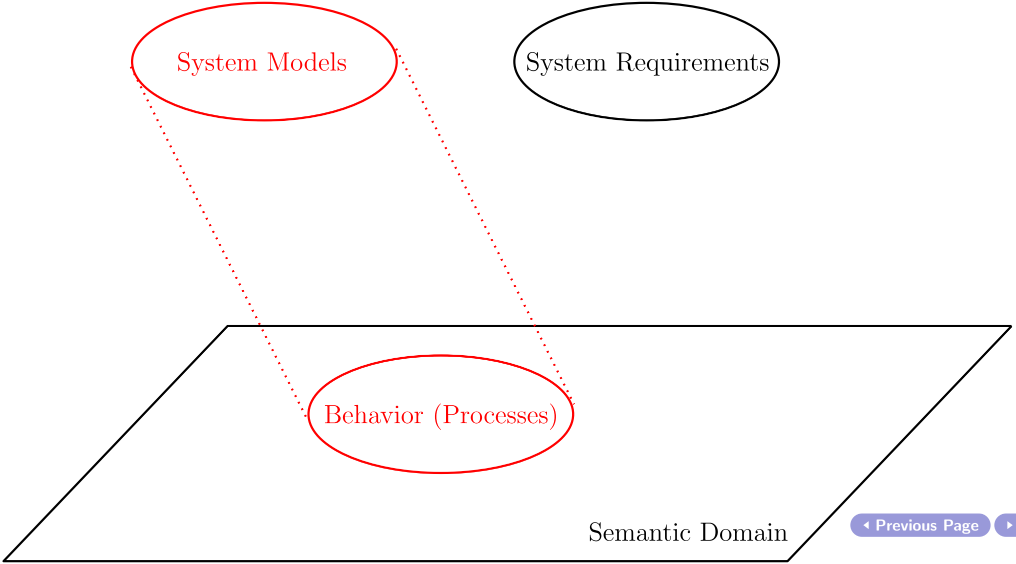


General Overview



Thank you very much.