Abstract—Formal methods have recently been successfully applied to construct verifiable high-level robot control. Most approaches use a discrete abstraction of the underlying continuous domain, and make simplifying assumptions about the physical execution of actions given a discrete implementation. Relaxing these assumptions unearths a number of challenges in the continuous implementation of automatically-synthesized hybrid controllers. This paper describes a controller-synthesis framework that ensures correct continuous behaviors by explicitly modeling the activation and completion of continuous low-level controllers. The synthesized controllers exhibit desired properties like immediate reactivity to sensor events and guaranteed safety of physical executions. The approach extends to any number of robot actions with arbitrary relative timings.

I. INTRODUCTION

Formal methods tools have recently been applied to the construction of controllers for high-level autonomous robot behaviors such as search and rescue missions and autonomous vehicle control [1], [4], [5], [7], [13]. Traditionally, the high-level control for such tasks is hard-coded, combining low-level techniques in an ad hoc fashion. Such an implementation provides no guarantees of fulfilling the desired requirements, motivating the use of formal methods to construct controllers that do provide such guarantees.

One technique that has been successfully applied to high-level robot planning is synthesis, in which a correct-by-construction controller is automatically synthesized from a formal task specification [7], [13]. These synthesis-based approaches operate on a discrete abstraction of the robot workspace and a formal specification of the environment assumptions and desired robot behavior, and produce an automaton fulfilling the specification on this abstraction if one exists. This automaton is used to construct a hybrid controller that calls low-level continuous controllers to execute each discrete transition. Consequently, a single transition between discrete states may correspond to the simultaneous execution of several low-level controllers.

In general, a robot with multiple action capabilities will use low-level controllers that take varying amounts of time to complete. This gives rise to a number of challenges in the continuous implementation of the synthesized hybrid controller. Using the continuous execution paradigm described in [7], all controllers except motion between adjacent regions of the workspace (i.e. all “fast” actions) are assumed to have instantaneous execution. Given a discrete transition between two states with different locations, the motion controller for driving the robot between regions is activated first, and the remaining controllers are only activated (or deactivated) once the robot has crossed into the new region. However, as described in [11], there are artifacts of this continuous execution paradigm that are not modeled at the discrete level. These include a delayed response to events in the environment, and unsafe continuous executions even though the discrete automaton is provably correct.

If the execution paradigm is changed to activate the fast actions as soon as possible (before the slow actions complete), the synthesis algorithm must be changed to ensure safe continuous execution [11]. However, with this approach, the execution paradigm ignores changes in the environment once a transition has been started, until the destination state is reached. The resulting controllers are correct under the assumption that the environment does not change during the execution of a discrete transition; correctness is thus at the expense of being fully responsive to the environment.

This paper presents a new approach that ensures both immediate reactivity and continued responsiveness to the environment, for low-level controllers of unknown relative execution times. Like the approach in [11], which changed the original synthesis algorithm [9] to ensure that fast actions could complete safely before slow ones, the approach presented here changes the algorithm (albeit in an entirely different manner) to ensure safe executions while maintaining tractability. Additionally, each robot action is explicitly separated into two events: the activation of the corresponding low level controller, and the triggering of a new sensor that indicates completion of its execution. This approach is shown to solve both the aforementioned problems.

This is one of the first works to consider the safety and correctness of continuous executions of synthesized automata arising from the physical nature of the problem domain. There are a few previous approaches that tried to ensure that the provably correct discrete controller results in correct continuous executions. For example, the authors of [8] resynthesized portions of the high-level controller at execution time when changes in the physical domain make the controller execution infeasible. In contrast, this work incorporates the continuous nature of the physical execution in the synthesis process from the beginning, by modeling the
non-instantaneous nature of controller execution.

II. BACKGROUND

This section provides an overview of the construction of provably correct robot controllers. The presented approach is implemented in the Linear Temporal Logic Mission Planning (LTLMoP) toolkit [3], which allows a specification written in a structured English grammar [6] to be transformed into a hybrid controller for use with real robots and in simulation. Example 1, adapted from [11], will serve to demonstrate the stages of synthesis for a simple high-level task.

A. Linear Temporal Logic (LTL)

Syntax: LTL formulas are defined recursively as:

\[ \varphi ::= \pi \mid \neg \varphi \mid \varphi \lor \varphi \mid \circ \varphi \mid \varphi^\bullet \varphi, \]

where \( \pi \in AP \) (a set of atomic propositions), \( \neg \) is negation, \( \lor \) is disjunction, \( \circ \) is “next”, and \( U \) is a strong “until”. Conjunction (\( \land \)), implication (\( \Rightarrow \)), equivalence (\( \Leftrightarrow \)) “eventually” (\( \circ \)) and “always” (\( \square \)) are derived operators.

Semantics: The truth of an LTL formula is evaluated over infinite sequences of states, corresponding to executions of a finite state machine representing the system. A state is an assignment of truth values to all propositions \( \pi \). The reader is referred to [2] for a formal definition of the semantics.

B. Discrete Abstraction and Formal Specification

The relevant features of the continuous robot control problem are abstracted using a finite set of propositions consisting of:

- \( \pi_s \) for every sensor input \( s \) (e.g., \( \pi_{person} \) is true if and only if a person is sensed)
- \( \pi_a \) for every robot action \( a \) (e.g., \( \pi_{camera} \) is true if and only if the robot has turned on the camera)
- \( \pi_r \) for every location or region \( r \) (e.g., \( \pi_{bedroom} \) is true if and only if the robot is in the bedroom).

The set of sensor propositions is denoted by \( \mathcal{X} \), and the set of action and location (i.e., robot-controlled) propositions by \( \mathcal{Y} \). \( \mathcal{L} \subseteq \mathcal{Y} \) denotes the set of location propositions. Thus \( \pi_s \in \mathcal{X}, \pi_a \in \mathcal{L}, \pi_r \in \mathcal{Y} \setminus \mathcal{L} \). The value of each \( \pi \in \mathcal{X} \cup \mathcal{Y} \) is the abstracted binary state of a low-level black box component, such as a thresholded sensor value or the robot’s location with respect to some partitioning of the workspace.

Example 1 Consider a simple two-room workspace where the two adjacent locations are labeled \( r_1 \) and \( r_2 \) (corresponding to \( \pi_r, 1 \) and \( \pi_r, 2 \)). The robot has one sensor, which senses a person (represented by proposition \( \pi_{person} \)), and one action, which is to turn a camera on or off (\( \pi_{camera} \)). The robot starts in room \( r_1 \) with the camera off. When it senses a person, it must turn on the camera. Once the camera is on, it must stay on. Finally, the robot must visit room \( r_2 \) infinitely often.

Here \( \mathcal{X} = \{ \pi_{person} \} \) and \( \mathcal{Y} = \{ \pi_{r, 1}, \pi_{r, 2}, \pi_{camera} \} \). A high-level task is specified on this discrete abstraction using an LTL formula over \( \mathcal{X} \cup \mathcal{Y} \). Two types of properties are allowed: safety properties, which guarantee that “something bad never happens,” and liveness conditions, which state that “something good (eventually) happens.” This work considers tasks that can be specified using formulas of the form \( \varphi_e \Rightarrow \varphi_r \), where \( \varphi_e \) represents the desired robot behavior, and \( \varphi_r \) encodes assumptions on the environment, including events external to the robot as well as the robot’s internal state, as perceived by its sensors. \( \varphi_e \) and \( \varphi_r \) each contain initial conditions (\( \varphi_{e, i} \), \( \varphi_{r, i} \)), safety requirements (\( \varphi_{e, f} \), \( \varphi_{r, f} \)) and liveness conditions, which state that \( \varphi_{e, f} \) (“something good (eventually) happens.”

Since the robot can be in exactly one location at any given time, the formula \( \varphi_r = \pi_{r, 1} \land \bigwedge_{r' \neq r} \neg \pi_{r, 2} \) is used to represent the robot being in region \( r \). The robot’s motion in the workspace is governed by adjacency of regions, and the availability of controllers to drive it between adjacent regions. In LTLMoP, the adjacency relation is automatically encoded as a logic formula \( \varphi_{trans} \). The adjacency relation for Example 1 is

\[ \varphi_{trans} = \square(\pi_{r, 1} \Rightarrow \circ \pi_{r, 1} \lor \pi_{r, 2}) \land \square(\pi_{r, 2} \Rightarrow \circ \pi_{r, 2} \lor \pi_{r, 1}). \]

The task in Example 1 corresponds to the following LTL specification:

\[
(\pi_{r, 1} \land \neg \pi_{camera}) \quad \#Initial
\]

- (Robot starts in region \( r_1 \) with the camera off)

\[
\land \square(\circ \pi_{person} \Rightarrow \circ \pi_{camera}) \quad \#Safety
\]

- (Activate the camera if you see a person)

\[
\land \square((\pi_{camera} \Rightarrow \circ \pi_{camera})) \quad \#Safety
\]

- (Camera stays on once turned on)

\[
\land \square(\pi_{r, 3}) \quad \#Liveness
\]

- (Go to \( r_2 \) infinitely often)

C. Synthesis

An LTL formula \( \varphi \) is realizable if there exists a finite state strategy that, for every infinite sequence of truth assignments to sensor propositions, gives an assignment to the robot propositions such that every infinite sequence of truth assignments to both sets of propositions generated in this manner satisfies \( \varphi \). The synthesis problem is to find a finite state machine encoding this strategy, i.e. whose executions satisfy \( \varphi \).

Synthesizing an automaton that realizes an arbitrary LTL formula is doubly exponential in the size of the formula [10]. When restricted to LTL formulas of the form \( \varphi_e \Rightarrow \varphi_r \), described above, the algorithm introduced in [9] permits synthesis in time polynomial in the size of the state space, while still capturing a large number of tasks specified in practice. Fig. 1 depicts the automaton synthesized for the above specification using this synthesis algorithm. Each state of the automaton is labeled with the subset of location and action propositions that are true in that state, and each tran-
Fig. 1: Synthesized automaton for Example 1. States are labeled with the location and action propositions that are true in that state. Transitions are labeled with the sensor propositions that are true for that transition to be enabled.

sition is labeled with the sensor propositions that are true for that transition to be enabled. Incoming transitions therefore also determine the truth value of the sensor propositions for each state. The labels $r_1, \text{camera}$ and $\text{person}$ represent $\pi_{r_1}, \pi_{\text{camera}}$ and $\pi_{\text{person}}$ respectively.

### III. CONTINUOUS EXECUTION

When a specification is realizable, synthesis yields an automaton that implements the specification in a discrete abstraction of the problem. If no such automaton exists, the user is presented with information about the cause of the unrealizability [12]. If an automaton is obtained, a controller that implements the corresponding continuous behavior is constructed by viewing the automaton as a hybrid controller, with a transition between two states achieved by the activation of one or more low-level continuous controllers corresponding to each robot proposition.

This section describes two approaches to physical execution of the synthesized automaton, and the challenges and shortcomings of each. Consider the workspace and task described in Example 1. Suppose the robot starts in room $r_1$, with its camera turned off and no persons sensed (so it is in the initial state $q_0$ in Fig. 1). Suppose it then senses a person. The safety condition $\square(\Diamond \pi_{\text{person}} \Rightarrow \Diamond \pi_{\text{camera}})$ requires it to turn on the camera. In order to fulfill its patrol goal, it will also try to go to room $r_2$. So the discrete transition in the automaton generated by the synthesis algorithm in [9] will be to state $q_1$. To execute the transition $(q_0, q_1)$ at the continuous level, a motion controller and a controller for turning on the camera must both be invoked.

#### A. Assuming Instantaneous Actions Except Motion [7]

Under the continuous execution paradigm in [7], all controllers except motion between adjacent regions (i.e. all fast actions) are assumed to have instantaneous execution. Given a discrete transition between two states with different locations, the motion controller for driving the robot between regions is activated first, and the remaining controllers only activated (or deactivated) once the robot has crossed into the new region. Thus, to execute the transition $(q_0, q_1)$, the hybrid controller constructed for Example 1 first activates the controller for moving from $r_1$ to $r_2$, and only once that boundary has been crossed will it activate the (instantaneous) controller for turning on the camera, completing the discrete transition $(q_0, q_1)$. This leads to a perceived delay in the robot turning on the camera in response to seeing a person.

#### B. Assuming Slow and Fast Actions [11]

Under the approach in [11], on the other hand, the robot’s actions are grouped into two sets based on execution duration, and all actuator controllers are executed simultaneously. Thus, to execute the transition $(q_0, q_1)$, the hybrid controller constructed for Example 1 activates the controller for turning on the camera (fast) simultaneously with that for moving from $r_1$ to $r_2$ (slow). The transition $(q_0, q_1)$ is completed only when the motion completes. However, the execution paradigm ignores changes in the environment once a transition has begun (i.e. following activation of a fast controller), until the destination is reached. This approach therefore produces controllers that are correct under the assumption that the environment does not change during the execution of a discrete transition, and will ignore any such changes.

Note that in this execution, the camera will turn on before the motion completes, and the robot is still in $r_1$, effectively resulting in an intermediate discrete state. To ensure safety of the continuous execution, the synthesis algorithm was changed to ensure that all such intermediate states are safe [11]. The automaton produced by the modified synthesis algorithm contains only those transitions for which the intermediate states are safe.

### IV. PROBLEM FORMULATION

The chief drawback of the continuous execution paradigm in III-A is delayed reactivity. It is usually desirable to respond to sensor inputs in a “greedy” manner, i.e. as soon as possible; for example, the camera should be turned on as soon as a person is sensed, regardless of the other actions to be performed. However, with the approach to continuous execution in III-A, the robot will not turn on its camera until the transition to $r_2$ has been completed. As described in III-B, this approach may also result in unsafe continuous executions even though the discrete automaton is provably correct.

Furthermore, during the transition $(q_0, q_1)$, if the person disappears before the motion completes, the transition is aborted, and the execution returns to $q_0$ (and then to an alternate successor state according to the automaton). Note that this causes the person to be ignored completely. On the other hand, if the camera is activated simultaneously with the motion, execution will pass through an intermediate state in which the fast action (camera) has completed but the slow action (motion) has not. If the robot safety formula required that the camera never be turned on in $r_1$, the execution resulting from simultaneous activation of the controllers would be unsafe even though the discrete solution is safe.

The correctness of the controllers generated in III-B, in comparison, relies on the assumption that the environment does not change during the execution of a discrete transition. Consider again the transition $(q_0, q_1)$ in Fig. 1 where the camera will be turned on at the same time as the motion but will complete before the robot has reached $r_2$. Suppose the robot stops sensing a person after the camera has been turned on, but before reaching $r_2$. Then the transition $(q_0, q_1)$ will be aborted, and $(q_0, q_2)$ will be taken instead. This results in the
camera going from on to off, violating the safety condition that enforces persistence of the camera action.

In the above example, the execution paradigm proposed in III-B will ignore the disappearance of a person after the camera has turned on. Correctness is therefore at the expense of being fully responsive to the environment during the time taken to move between regions. Additionally, the approach in III-B assumes a known ordering on action completion times, reducing the number of intermediate states to be checked; it extends to an unknown ordering with an exponential blow-up in the number of intermediate states considered.

The shortcomings of the two existing approaches lead to the following problem, which this paper solves.

**Problem 1** Given a specification $\phi$ and a set of actions with unknown relative completion times, construct an automaton such that every continuous execution satisfies $\phi$ while allowing immediate reactivity as well as continual responsiveness to changes in the environment.

V. Solution

This section proposes a solution to Problem 1 for arbitrary (but finite) action completion times. To account for the non-instantaneous execution of continuous controllers, each robot action is viewed as the activation of the corresponding low level controller, and a new sensor proposition is introduced in the discrete model to indicate whether the controller has completed execution.

A. Discrete Abstraction

The set of propositions is now modified to include:

- $\pi_r$ for each sensor input $s$ (e.g., $\pi_{person}$ is true if and only if a person is sensed)
- $\pi_a$ for the activation of each robot action $a$ (e.g., $\pi_{camera}$ is true if and only if the robot has activated the controller to turn on the camera). Similarly, $\neg \pi_a$ represents the activation of the controller for turning $a$ off.
- $\pi_r$ for the initiation of motion towards each region $r$ (e.g., $\pi_{bedroom}$ is true if and only if the robot is trying to move to the bedroom). $\phi_r$ is defined as in Section II.
- $\pi^+_r, \pi^-_r$ for the completion of the controller for turning action $a$ on, or motion to region $r$ (e.g., $\pi^-_{bedroom}$ is true if and only if the robot has arrived in the bedroom, and $\pi^+_{camera}$ is true if and only if the camera has finished turning on). $\neg \pi^+_r$ represents the completion of the controller for turning action $a$ off.

Action/motion completion is modeled as an event sensed by the robot, and therefore $X = \{ \pi^+_r \} \cup \{ \pi^+_a \} \cup \{ \pi_r \}$, $Y = \{ \pi_a \} \cup \{ \pi_r \}$. For Example 1, $X = \{ \pi_{person}, \pi^+_r, \pi^-_r, \pi^+_{camera} \}$ and $Y = \{ \pi_r, \pi^+_r, \pi^-_r \}$.

B. Formal Specification Transformation

Given this discrete abstraction, the task specification now governs which actions can be activated by the robot, and how the action-completion sensors behave.

1) Proposition replacement in original specification: A specification $\phi = (\phi_r \Rightarrow \phi_a)$ in [7] is modified as follows:

- Initial conditions specify the sensed state of the robot, so every occurrence of $\pi^+_a$ in $\phi_a$ is replaced with $\pi^+_a$.
- Robot goals are predicated on the completion of actions (as sensed by the corresponding sensor). So every occurrence of $\pi^+_r$ in $\phi_r$ is replaced with $\pi^+_r$. Similarly, robot goals refer to the sensed location $\pi^+_r$ rather than just the activation of the motion controller $\pi_r$.
- Robot safety conditions govern which controllers are to be activated in response to events in the environment, and may refer to the sensed location of action completion as well as events in the environment. The user input language, such as that presented in [6], must therefore allow distinguishing between a reference to $\pi_a$ and $\pi^+_a$ in safety conditions, as discussed in VI-B.

The resulting LTL specification is denoted $\phi' = \phi^+_a \Rightarrow \phi_a$. Table I presents the LTL formulas corresponding to the specification for Example 1, provided in Section II, before and after proposition replacement.

2) Robot Transition Relation: The allowed robot motion now depends on the sensed location. Given a region $r$, let $Adj(r)$ denote the regions adjacent to $r$ (including $r$ itself). $\phi_{trans}$ II-B then changes as follows:

$$ \phi_{trans}' = \bigwedge_{r} \square(\pi^+_a \Rightarrow \bigvee_{r' \in Adj(r)} \phi_{r'}) $$

For Example 1, $\phi_{trans}' = \square(\pi^+_a \Rightarrow (\bigwedge_{r} \phi_{r} \vee \bigwedge_{r} \phi_{r}))$.

3) Sensor Assumptions: Additional assumptions define the effects of the robot activating its various controllers.

$$ \phi_{New} = \square(\pi^+_a \Leftrightarrow \bigwedge_{r \neq r'} \square(\pi^+_a \land \pi_r \Rightarrow (\bigwedge_{r \in Adj(r)} \pi^+_a \vee \bigwedge_{r \in Adj(r)} \pi^+_a)) \land \bigwedge_{a} \bigwedge_{r} \bigwedge_{r'}(\pi^+_a \land \pi_r \Rightarrow \bigwedge_{r} \pi^+_a) \land \bigwedge_{a} \bigwedge_{r} \bigwedge_{r'}(\neg \pi^+_a \land \pi_r \Rightarrow \bigwedge_{r} \pi^+_a) $$

Conjunct (1) enforces mutual exclusion between the physical locations of the robot. Conjunct (2) governs how the robot’s location can change in a single time step in response to the activation of the motion controllers. Conjuncts (3–4) govern the completion of other actions in response to the activation of the corresponding controllers. In Example 1,

$$ \phi_{New} = \square(\pi^+_a \Leftrightarrow \pi_{camera}) \land \bigwedge_{a}(\pi_{camera} \Leftrightarrow \pi_{camera}) \land \bigwedge_{a}(\pi_{camera} \Leftrightarrow \pi_{camera}) $$

Note that this paper considers actions other than motion to have on and off modes only, but the approach extends to other types of actions.
TABLE I: Replacing propositions in the task specification for Example 1

<table>
<thead>
<tr>
<th>English specification</th>
<th>Original LTL ($\varphi$)</th>
<th>New LTL ($\varphi'$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robot starts in region $r_1$ with the camera off</td>
<td>$\varphi_1 \land \neg \pi_{camera}$</td>
<td>$\varphi'<em>1 \land \neg \pi</em>{camera}$</td>
</tr>
<tr>
<td>Activate the camera if you see a person</td>
<td>$\Box (\lozenge \chi_{person} \Rightarrow \lozenge \pi_{camera})$</td>
<td>$\Box (\lozenge \chi_{person} \Rightarrow \lozenge \pi_{camera})$</td>
</tr>
<tr>
<td>Camera stays on once turned on</td>
<td>$\Box (\pi_{camera} \Rightarrow \lozenge \pi_{camera})$</td>
<td>$\Box (\pi_{camera} \Rightarrow \lozenge \pi_{camera})$</td>
</tr>
<tr>
<td>Go to $r_2$ infinitely often</td>
<td>$\Box \lozenge (\pi_2)$</td>
<td>$\Box \lozenge (\pi_2)$</td>
</tr>
</tbody>
</table>

For example, conjunct (7) states that if the robot is in $r_1$ (i.e. $\pi^c_1$ is true) and is activating the controller to move to $r_2$ (i.e. $\varphi_2$), then in the next time step, the robot is either still in $r_1$ ($\pi^c_1$ is true) or has reached $r_2$ ($\pi^c_2$ is true). Conjunct (10) states that if the camera is already on and is supposed to be on, it will stay on.

4) Fairness conditions: In addition to the above safety conditions, additional constraints on the environment are required to ensure that every action/motion eventually completes, i.e. that the robot’s environment is in some sense “fair”. A naive approach is to add an environment assumption that every controller activation or deactivation eventually results in completion, i.e. the fairness conditions $\Box \Diamond (\pi_2 \Rightarrow \lozenge \pi^c_1)$ and $\Box \Diamond (\neg \pi_2 \Rightarrow \lozenge \neg \pi^c_1)$ for every action $a$.

However, this adds two fairness assumptions to the specification for every action. Since the synthesis algorithm scales linearly with the number of propositions, it is important to minimize added assumptions. This is achieved by introducing a single fairness condition that incorporates the possibility that the robot is forced to “change its mind” by events in the environment. To this end, two new Boolean formulas $\varphi^c_a$ and $\varphi^a_a$ are defined for each action $a$:

$\varphi^c_a = (\pi_a \land \Box \pi^c_a) \lor (\neg \pi_a \land \neg \Box \pi^c_a)$

$\varphi^a_a = (\pi_a \land \neg \Box \pi_a) \lor (\neg \pi_a \land \Box \pi_a)$

$\varphi^c_a$ holds when the activation (or de-activation) of the controller for $a$ has completed execution. $\varphi^a_a$ captures the robot changing its mind (such as by toggling the camera). A single pair of formulas $\varphi^c_{loc}$, $\varphi^a_{loc}$ suffices for motion since locations are mutually exclusive and the robot cannot try to move to two locations at once:

$\varphi^c_{loc} = V_r (\pi_r \land \Box \pi^c_r)$,

$\varphi^a_{loc} = V_r (\pi_r \land \neg \Box \pi_r)$

The complete fairness assumption added is:

$\varphi^f_{fair} = \Box \Diamond (\varphi^c_{loc} \lor \varphi^a_{loc})$

Note that every execution satisfying both fairness conditions described above for activation and deactivation also satisfies this fairness condition. Moreover, there is only one such assumption added for each action $a$ (in Example 1, there is one such assumption for the camera). Additionally, there is one assumption $\varphi^f_{fair}$ for motion. For Example 1,

$\varphi^f_{camera} = \Box \Diamond (\pi_{camera} \land \Box \pi_{camera}) \lor (\neg \pi_{camera} \land \neg \Box \pi_{camera})$

$\varphi^f_{loc} = \Box \Diamond (\pi_r \land \Box \pi^c_r) \lor (\pi_r \land \neg \Box \pi^c_r)$

Given a task specification $\varphi = (\varphi_e \Rightarrow \varphi_i)$, the LTL specification used to synthesize a controller (after proposition replacement, changing the robot transition relation, and adding sensor assumptions and fairness conditions) is now

$\varphi_{new} = \varphi'_e \land \varphi_{New} \land \bigwedge_a \varphi^c_a \land \varphi^a_{loc} \Rightarrow \varphi'_i \land \varphi^f_{trans}$

C. Synthesis

Since the formulas $\varphi^c_a$ and $\varphi^a_a$ in the proposed liveness condition $\varphi^c_a$ govern both current and next time steps, the original synthesis algorithm in [9] cannot be applied as-is to synthesize an implementing automaton for the specification. Liveness conditions that incorporate temporal formulas with both current and next time steps are handled by changing the computation of the set of robot-winning strategies in the synthesis algorithm [9].

Note that it is possible to use the original synthesis algorithm (which only allows simple Boolean formulas in liveness conditions) to synthesize a controller by introducing a new proposition, $\pi^c_{completion,change}$, and the safety condition

$\Box (\pi^c_{completion,change} \Leftrightarrow (\varphi^c_{completion} \lor \varphi^a_{change}))$

This allows the additional liveness to instead be written as

$\varphi^f_{fair} = \Box \Diamond \pi^c_{completion,change}$

However, this introduces one new proposition per robot action. The complexity of the synthesis algorithm scales with the size of the state space, which in turn scales exponentially with the number of propositions. In addition, extra propositions result in larger automata that are harder to read.

Even with the above change to the synthesis algorithm, one environment proposition must be added per robot action (corresponding to the sensor for action completion). In the worst case, the time taken for synthesis is therefore still increased by a factor of $2^{|\mathcal{P}|}$ over the original approach. With the original synthesis algorithm, the increase is by a factor of $4^{|\mathcal{P}|}$ (since two new propositions are required per action).

D. Continuous Execution

Given a state $q$, observed sensor values $X \subset \mathcal{X}$ and the corresponding next state $q'$ in the automaton, the transition $(q, q')$ is executed by simultaneously invoking the controllers corresponding to every action or location proposition $\pi_a$ that changes value from $q$ to $q'$. Note that the current sensed state of the system, as represented by $X$, determines which actions $a$ can be activated in $q'$. Transitions in the automaton are instantaneous, as they correspond to activation or deactivation of controllers, but the controllers themselves may take several discrete transitions to complete execution. The resulting controller exhibits the desired properties:

- actions are executed immediately in response to sensor events, eliminating the delayed reactivity of [7]
• safety of continuous executions is guaranteed even when
  the robot changes its mind
• the approach extends to any number of robot actions,
  with arbitrary relative timings, although the computa-
  tional burden increases for a large number of actions.

VI. TEST CASES

This section provides test cases illustrating the effectiveness of the proposed solution. The robot controllers for the examples presented were synthesized using LTLMoP.

A. Safety of physical execution

Fig. 3 depicts an excerpt of the automaton synthesized for Example 1. The full automaton has 7 states and is omitted for conciseness. Negated sensor labels are omitted from the transitions for clarity. The label \( c_x \) represents the fact that the camera is being turned on even though the person has disappeared.

In state \( q_0 \), the robot is in \( r_2 \) (as indicated by \( \pi_{r_2} \) being true on the incoming transitions) with the camera off \( (\pi_{\text{camera}} \) is false), and is trying to go to \( r_1 \) (as indicated by the action \( \pi_{r_1} \) being true). Consider the transition \( (q_0, q_1) \). The robot reaches \( r_1 \) (indicated by \( \pi_{r_1} \) being true on the transition into \( q_1 \). It is now trying to go to \( r_2 \), indicated by \( \pi_{r_2} \). In \( q_1 \), if the robot does not sense a person, it either moves back to \( q_0 \) or stays in \( q_1 \) depending on whether it has reached \( r_2 \) yet. On the other hand, suppose the robot senses a person in \( q_1 \) before it has reached \( r_2 \), the execution moves to state \( q_3 \), where the robot is still trying to get to \( r_2 \) but is now additionally activating the camera. On the other hand, the transition \( (q_1, q_2) \) is taken if the robot senses a person at the same time as it reaches \( r_2 \) (as indicated by sensor proposition \( \pi_{r_2} \) being true on that transition).

Note that in the continuous execution of the above automaton, all the controllers are invoked at the same time. For example, when transitioning from \( q_1 \) (where the robot is in \( r_1 \)) to \( q_3 \), the controller for moving to \( r_2 \) and for turning on the camera are being activated simultaneously. Any difference in their completion times is captured by the corresponding sensor propositions. Even if the person disappears before the motion from \( r_1 \) to \( r_2 \) is completed, the transition \( (q_1, q_3) \) is still taken (followed by a transition out of \( q_3 \) that corresponds to the person not being seen), and the camera is still being activated in \( q_3 \). This ensures that the person is not ignored, since there is an explicit state representing the fact that the camera is being turned on even though the person has disappeared.

B. Activation and Completion Dependent Safety Properties

Desired safety properties can now also be defined in terms of which actions have completed, rather than which actions are activated. For example, when requiring that the camera not be turned on in \( r_1 \), the added system safety can be either

\[ \square(\neg \pi_{r_1} \Rightarrow \neg \pi_{\text{camera}}) \] or \[ \square(\pi_{r_1} \Rightarrow \neg \square \pi_{\text{camera}}) \].

The first of these enforces the requirement that the camera not physically be on while the robot is still in \( r_1 \), whereas the second states that the controller for turning on the camera will not be activated while the robot is in \( r_1 \). Note that both specifications will produce the same observed behavior, but the first one allows the robot to try to turn on the camera in \( r_1 \), whereas the second does not.

Recall from Section IV the challenge of ensuring safe execution when transitions are aborted due to changes in the environment. The safety condition enforcing that the camera stays on once turned on (in Example 1) can be expressed in terms of controller activation and completion, as either

\[ \square(\pi_{\text{camera}} \Rightarrow \neg \square \pi_{\text{camera}}) \] or \[ \square(\pi_{\text{camera}} \Rightarrow \neg \square \pi_{\text{camera}}) \].

The first of these prevents the robot from toggling \( \pi_{\text{camera}} \) (e.g., by flicking the camera switch on and off). The second prevents the robot from trying to turn the camera off once it has completely turned on; toggling \( \pi_{\text{camera}} \) is allowed until the camera has actually turned on. Similarly, the specification can be fine-tuned to distinguish between continuous controllers that can be aborted and those that cannot.

Example 2 Consider the workspace depicted in Fig. 2(a). The robot starts in \( r_1 \) and has to visit \( r_4 \). However, if it sees a stop sign in either \( r_2 \) or \( r_3 \), it cannot pass through that room. A safety assumption on the environment guarantees that there will never be stop signs in both \( r_2 \) and \( r_3 \) at the same time. There are initially no stop signs.

Given this specification, one might expect an implementing controller that drives the robot from \( r_1 \) to \( r_4 \) via whichever room \( (r_2 \) or \( r_3 \)) does not have a stop sign. However, if a stop sign appears while the robot is driving to this room, the robot has to turn around and try the other room. If this happens every time the robot starts moving towards \( r_4 \), it will never be able to reach \( r_4 \). With the approach in [7], the specification for Example 2 is:

\[
\begin{align*}
\land \neg \pi_{\text{stop sign in } r_2} \land \neg \pi_{\text{stop sign in } r_3} & \quad \text{#Env Initial} \\
\land \neg [\neg \pi_{\text{stop sign in } r_2} \land \neg \pi_{\text{stop sign in } r_3}] & \quad \text{#Env Safety} \\
\implies & \\
\phi_{r_1} & \quad \text{#Robot Initial} \\
\land \neg [\neg \pi_{\text{stop sign in } r_2} \iff \neg \pi_{r_2}] & \quad \text{#Robot Safety} \\
\land \neg [\neg \pi_{\text{stop sign in } r_3} \iff \neg \pi_{r_3}] & \quad \text{#Robot Safety} \\
\land \square \pi_{r_1} & \quad \text{#Robot Liveness} \\
\end{align*}
\]

This specification is realizable via the approach in [7], and the synthesized automaton is depicted in Fig. 2(b). However, consider what happens when the robot is in state \( q_0 \) (where it is in \( r_1 \)), and sees a stop sign in \( r_2 \). The robot will start to move towards \( r_3 \) (and state \( q_1 \)). Suppose that before the robot has entered \( r_3 \), the stop sign in \( r_2 \) disappears but one appears in \( r_3 \). The robot will abort the discrete transition \( (q_0, q_1) \) and start heading to \( r_2 \) to take the transition \( (q_0, q_2) \) instead. If the stop sign’s location changes faster than the robot can move, the robot will be trapped in \( r_1 \), because it will keep changing its mind between the above two discrete
transitions. This is therefore an example of a high-level task that produces a controller under the synthesis approach of [7], but whose physical execution does not accomplish the specified behaviour because of an inadequate modeling of the underlying physical system.

With the new discrete abstraction, task specification transformation and execution paradigm presented in this paper, the robot initial condition in the above specification changes to $\pi_{c r1}$, and the robot goal becomes $\square \Diamond (\pi_{c r4})$. This specification (with the additional formulas introduced in Section V) is unrealizable, and no automaton is obtained. As noted above, this the safer, more desirable outcome, since there exists an environment strategy that toggles the stop signs between $r_2$ and $r_3$ and prevents the robot from fulfilling the specification.

Recent work has tackled the question of analyzing specifications that are unrealizable [12]. Future research will explain cases of unrealizability arising from the relative execution times of the low-level controllers, and present users with this information in a useful manner.

VII. CONCLUSIONS

This paper describes a discrete abstraction, specification transformation, synthesis algorithm and execution paradigm that ensures correct continuous behaviors with immediate reactivity and continued responsiveness. By explicitly modeling the activation and completion of the continuous low-level controllers, the approach is able to handle actions of arbitrary relative execution times, solving problems that escape previous approaches. The synthesis algorithm is changed to lower the computational overhead of the controller construction. Future work includes analyzing specifications that are unrealizable under the new paradigm due to the relative execution times of the low-level controllers, and presenting users with this information in a useful manner.

REFERENCES