Boosting Automated Reasoning by Mining Existing Proofs

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L4.Verified

25-30 years combined effort

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Problem:

Finding a suitable sequence of proof steps is hard!

Proof Automation

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  - Reduces Human Intervention
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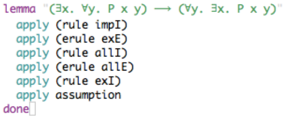
- Restricted by underlying logic
  - Expressivity vs Automation Tradeoff

Proof Libraries



```
lemma "(∃x. ∀y. P x y) ⟶ (∀y. ∃x. P x y)"
  apply (rule impI)
  apply (erule exE)
  apply (rule allI)
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  apply (rule exI)
  apply assumption
done
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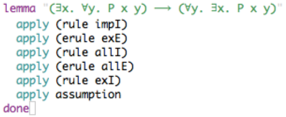

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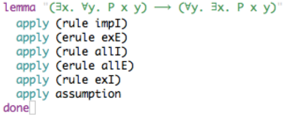

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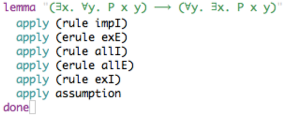

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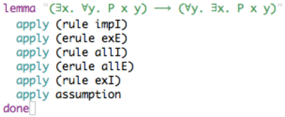

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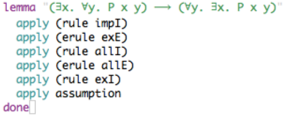

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Idea:

Can we use this information to automate new proofs?

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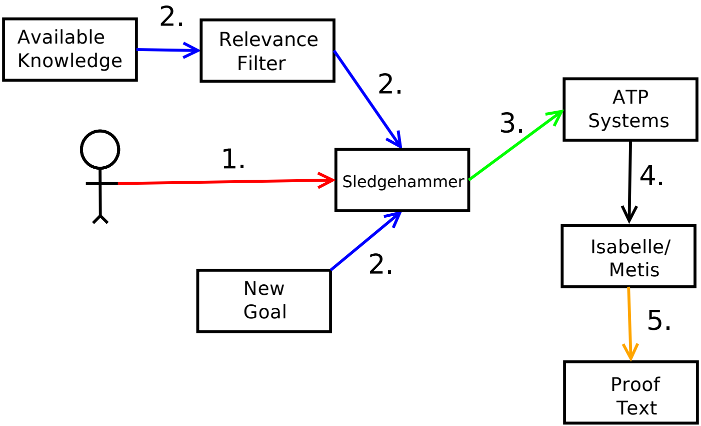

Increasing Automation in ITP's - Proof Hints

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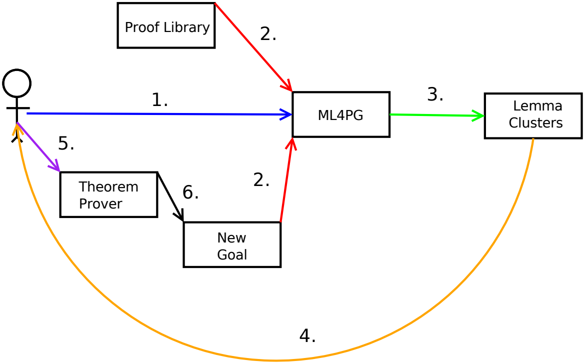

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Useful Sequences - Sequences of proof steps that could prove useful in proving some new goal

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Sequence 1:	**Sequence 2:**	**Tactic:**
rule impI assumption	rule conjI assumption	(rule impI OR rule conjI) THEN assumption

Previous Tactic Mining Work

Carried out by Hazel Duncan at Edinburgh.

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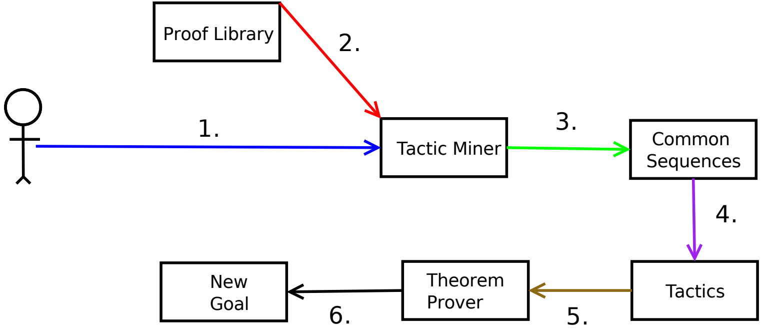

Critique of Duncan’s approach

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There are some limitations of Duncan's work:

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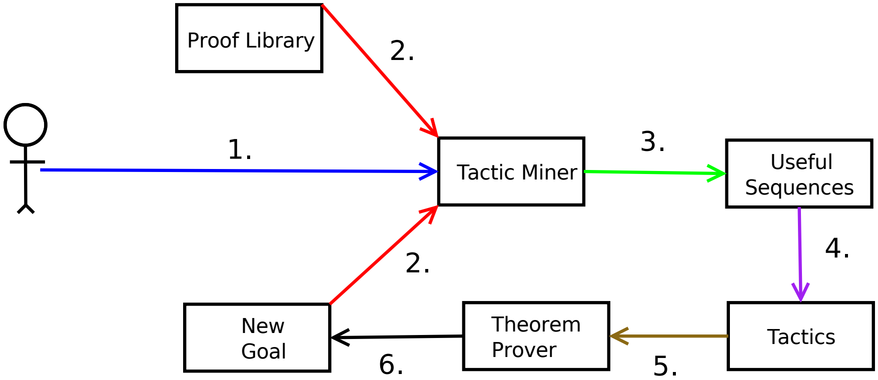
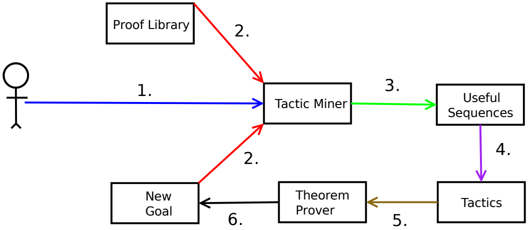
- No subgoal information
- Inefficent tactic application

My Tactic Mining Approach



1. How can we deal with complex Higher Order Languages?

Variable instantiations and proof directives



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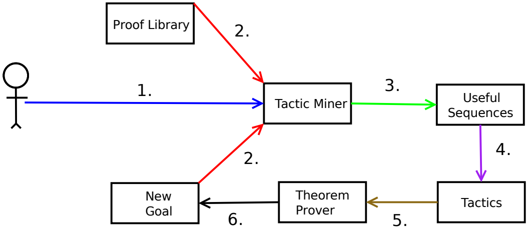

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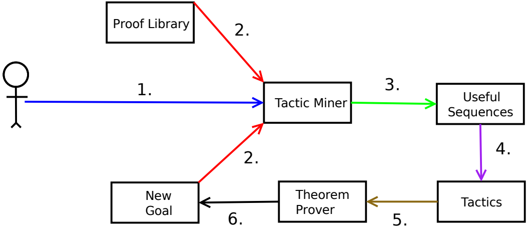
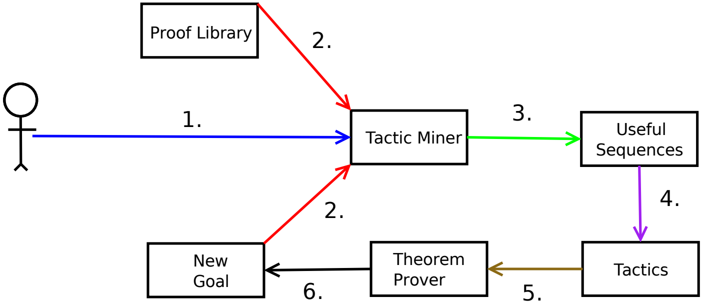
Different proof styles



2. Which Data Mining Techniques can help?

An open research question



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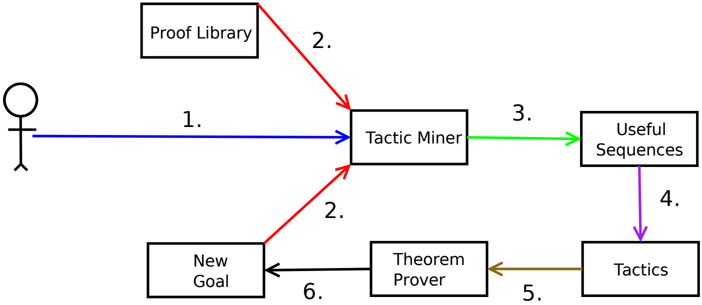
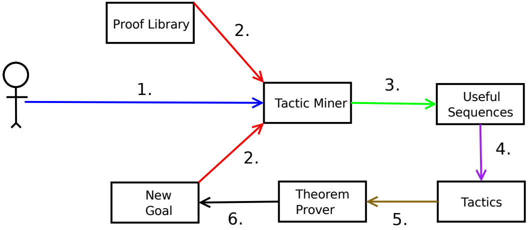
Two tasks: Finding the patterns and generalising into tactics



3. How will the theorem prover and tactic miner communicate?

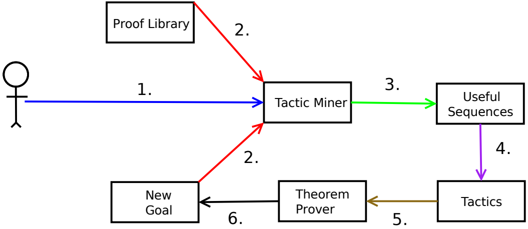
We require two methods of communication to be defined:

- Theorem Prover to Tactic Miner
- Tactic Miner to Theorem Prover



4. How can we make use of negative information?

Leverage negative information from:



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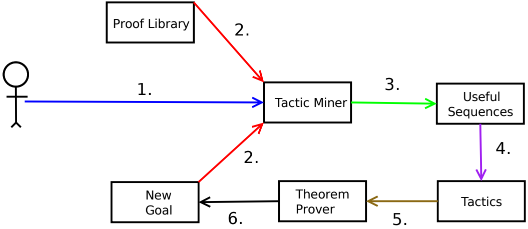

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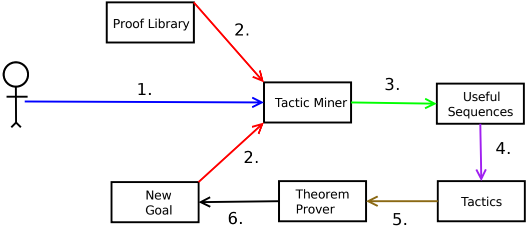

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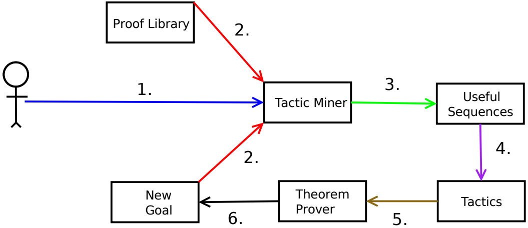
Would enable a supervised learning approach.



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- Data Extraction from Isabelle
- Considering learning techniques

Any Questions?

Please feel free to ask me any questions, either now or at any point during the workshop!