Queries, Modalities, Relations, Trees, XPath
Lecture VII
Core XPath and beyond

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July 2010: draft

Basic Axioms I: Idempotent Semirings

ISAx1	$(A \cup B) \cup C$	$\equiv$	$A \cup (B \cup C)$
ISAx2	$A \cup B$	$\equiv$	$B \cup A$
ISAx3	$A \cup A$	$\equiv$	A
ISAx4	$A/(B/C)$	$\equiv$	$(A/B)/C$
ISAx5	$\cdot/A$	$\equiv$	A
	$A/\cdot$	$\equiv$	A
ISAx6	$A/(B \cup C)$	$\equiv$	$A/B \cup A/C$
	$(A \cup B)/C$	$\equiv$	$A/C \cup B/C$
ISAx7	$\bot$	$\subseteq$	A

Distributive lattices, Kleene algebras, Tarski's relation algebras: they all have idempotent semiring reducts.
Idempotency is the axiom ISAx3.
$\bot$ abbreviates $\cdot[\neg\langle\cdot\rangle]$

Basic Axioms II: Predicate Axioms

PrAx1	$A\,[\neg\langle B\rangle]\,/B$	$\equiv$	$\bot$
PrAx2	$A\,[\phi \vee \psi]$	$\equiv$	$A\,[\phi] \cup A\,[\psi]$
PrAx3	$(A/B)\,[\phi]$	$\equiv$	$A/B\,[\phi]$
PrAx4	$\cdot\,[\langle\cdot\rangle]$	$\equiv$	$\cdot$

In Tarski's relation algebras and XPath 2.0,
predicates can be defined away
Note that PrAx3 would not be valid
if we allowed unrestricted positional predicates

Basic Axioms III: Node Axioms

NdAx1	ϕ	$\equiv$	$\neg(\neg\phi \vee \psi) \vee \neg(\neg\phi \vee \neg\psi)$
NdAx2	$\langle A \cup B \rangle$	$\equiv$	$\langle A \rangle \vee \langle B \rangle$
NdAx3	$\langle A/B \rangle$	$\equiv$	$\langle A\,[\langle B \rangle]\,\rangle$
NdAx4	$\langle \cdot\,[\phi]\,\rangle$	$\equiv$	ϕ

Note how little was needed to ensure booleanity!
(by Huntington's result from the 1930's)
And NdAx2–NdAx4 just mimick PrAx2—PrAx4
(redundancy: price to pay for two-sorted signature)

Axioms in one-sorted signature

Recall all the two-sorted axioms for predicates and expressions:

PrAx1	$A\,[\neg\langle B\rangle]\,/B$	$\equiv$	$\bot$
PrAx2	$A\,[\phi \vee \psi]$	$\equiv$	$A\,[\phi] \cup A\,[\psi]$
PrAx3	$(A/B)\,[\phi]$	$\equiv$	$A/B\,[\phi]$
PrAx4	$\cdot\,[\langle\cdot\rangle]$	$\equiv$	$\cdot$
NdAx1	ϕ	$\equiv$	$\neg(\neg\phi \vee \psi) \vee \neg(\neg\phi \vee \neg\psi)$
NdAx2	$\langle A \cup B\rangle$	$\equiv$	$\langle A\rangle \vee \langle B\rangle$
NdAx3	$\langle A/B\rangle$	$\equiv$	$\langle A\,[\langle B\rangle]\,\rangle$
NdAx4	$\langle\cdot\,[\phi]\,\rangle$	$\equiv$	ϕ

Axioms in one-sorted signature

Here is a one-sorted axiomatization for $\sim$ over idempotent semi-ring axioms found by Hollenberg:

$$
\begin{array}{lcl}
\sim A/A & \equiv & \bot \\
\sim\sim A/A & \equiv & A \\
\sim(A/B)/A & \equiv & (\sim(A/B)/A)/\sim B \\
\sim(A \cup B) & \equiv & \sim A/\sim B \\
\sim A \cup \sim B & \equiv & \sim\sim(\sim A \cup \sim B)
\end{array}
$$

We need to add one more axiom for tests:

$$?p \equiv \sim\sim ?p$$

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In order to find more interesting equivalences,
we have to move to other fragments

Axioms for Linear Axes

The non-transitive case:

LinAx1 $\mathsf{s}\,[\neg\phi] \quad \equiv \quad \cdot\,[\neg\langle \mathsf{s}\,[\phi]\,\rangle]\,/\mathsf{s}$ for $\mathsf{s} \in \{\rightarrow, \leftarrow, \uparrow\}$

This forces functionality of the corresponding axis

Axioms for Transitive Axes

One for node expressions, one for path expressions:

TransAx1	$\langle \mathsf{s}^+[\phi]\rangle$	$\equiv$	$\langle \mathsf{s}^+[\phi \wedge \neg\langle \mathsf{s}^+[\phi]\rangle]\rangle$
TransAx2	s^+	$\equiv$	$\mathsf{s}^+ \cup \mathsf{s}^+/\mathsf{s}^+$

The first one is called the Löb axiom and forces well-foundedness
Don't get modal logicians started on it—
people wrote books about this formula

In particular, all the consequences of TransAx2 for *node expressions*
can be already derived from TransAx1
I can neither prove nor disprove that for *path expressions*
TransAx2 is (ir-)redundant

Finally, Axes which Are Both Transitive and Linear

LinAx2 $\cdot [\langle s^{+} [\phi] \rangle] / s^{+} \quad \equiv \quad s^{+} [\phi] \cup s^{+} [\phi] / s^{+} \cup s^{+} [\langle s^{+} [\phi] \rangle]$
for $s \in \{\rightarrow, \leftarrow, \uparrow\}$

together with transitivity axioms

This forces the corresponding axis is a linear order

Single Axis Completeness Result

Theorem

- *Base axioms are complete for Core XPath($\downarrow$)*
- *Base axioms with LinAx1 are complete for other intransitive single axis fragments*
- *Base axioms with TransAx1 and TransAx2 are complete for Core XPath($\downarrow^{+}$)*
- *Base axioms with TransAx1, TransAx2 and LinAx2 are complete for other transitive single axis fragments*

A Few Words About Proofs

- First, rewrite node expressions to simple node expressions:

$$\text{siNode} ::= \langle\cdot\rangle \mid p \mid \langle \text{a}\,[\text{siNode}]\,\rangle \mid \neg\text{siNode} \mid \text{siNode}\vee\text{siNode}$$

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  They are isomorphic variants of modal formulas
- Using normal form theorems for modal logic, we provide a completeness proof for node expressions

A Few Words About Proofs cntd.

- Then we rewrite all path expressions as sums of sum-free expressions of the form

$$S = \cdot\,[\beta_1]\,/\mathsf{a}\,[\beta_2]\,/\ldots/\mathsf{a}\,[\beta_\ell]\,,$$

(all β_i are normal forms of

- the same nesting degree in case of transitive axes
- strictly decreasing degree for intransitive axes)

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  - one is a subsequence of the other—provably contained or
  - there is a countermodel for containment

Aside: the issue of labels

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But as our proofs used only Birkhoff's rules
they are quite flexible and adding this axiom does not hurt

Starting from the Other End

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LET'S GO FOR THE WHOLE CORE XPATH!

Axiom For Axes Dependencies

$$\text{TreeAx1}\begin{cases} s^{+}/s \cup s \equiv s^{+} \\ s/s^{+} \cup s \equiv s^{+} \end{cases}$$

$$\text{TreeAx2} \quad s[\phi]/s^{\smile} \equiv \cdot[\langle s[\phi]\rangle] \text{ (for s distinct than } \uparrow)$$

$$\text{TreeAx3} \quad \uparrow[\phi]/\downarrow \equiv (\leftarrow^{+} \cup \rightarrow^{+} \cup \cdot)[\langle\uparrow[\phi]\rangle]$$

$$\text{TreeAx4}\begin{cases} \leftarrow^{+} \equiv \leftarrow^{+}[\langle\uparrow\rangle] \\ \rightarrow^{+} \equiv \rightarrow^{+}[\langle\uparrow\rangle] \end{cases}$$

TreeAx1 says: s^{+} is a transitive closure of s
TreeAx2 says non-child axes are functional
and describes their converse
TreeAx3 forces $\uparrow$ is the converse of (non-functional) $\downarrow$
with TreeAx4, it also describes how horizontal and vertical axes interplay

Theorem

The axioms presented so far are complete for Core XPath node expressions

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Proof.

By reduction to simple node expressions
and derivation of all axioms of modal logic of finite trees
by Blackburn, Meyer-Viol, de Rijke □

(boolean axioms)

$$
\begin{array}{lcl}
\langle \mathsf{s}[\neg\langle\cdot\rangle]\rangle & \equiv & \neg\langle\cdot\rangle \\
\langle \mathsf{s}[\phi \vee \psi]\rangle & \equiv & \langle \mathsf{s}[\phi]\rangle \vee \langle \mathsf{s}[\psi]\rangle \\
\phi & \leq & \neg\langle \mathsf{s}[\neg\langle \mathsf{s}^{\smile}[\phi]\rangle]\rangle \\
\langle \mathsf{s}[\neg\phi]\rangle \wedge \langle \mathsf{s}[\phi]\rangle & \equiv & \neg\langle\cdot\rangle(\text{ for s distinct than } \uparrow) \\
\langle \mathsf{s}[\phi]\rangle \vee \langle \mathsf{s}[\langle \mathsf{s}^{+}[\phi]\rangle]\rangle & \equiv & \langle \mathsf{s}^{+}[\phi]\rangle \\
\neg\langle \mathsf{s}[\phi]\rangle \wedge \langle \mathsf{s}^{+}[\phi]\rangle & \leq & \langle \mathsf{s}^{+}[\neg\phi \wedge \langle \mathsf{s}[\phi]\rangle]\rangle \\
\langle \mathsf{s}[\langle\cdot\rangle]\rangle & \leq & \langle \mathsf{s}^{+}[\neg\langle \mathsf{s}[\langle\cdot\rangle]\rangle]\rangle \\
\text{TransAx1 for } \downarrow^{+} \text{ and } \rightarrow^{+} & & \\
\langle \downarrow[\neg\langle\leftarrow\rangle \wedge \neg\langle\rightarrow^{*}[\phi]\rangle]\rangle & \leq & \neg\langle \downarrow[\phi]\rangle \\
\langle \downarrow[\phi]\rangle & \leq & \langle \downarrow[\neg\langle\leftarrow\rangle]\rangle \wedge \langle \downarrow[\neg\langle\rightarrow\rangle]\rangle \\
\neg\langle\uparrow\rangle & \leq & \neg\langle\leftarrow\rangle \wedge \neg\langle\rightarrow\rangle
\end{array}
$$

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Add the separability rule:

(Sep) IF $\langle A\,[p]\,\rangle \equiv \langle B\,[p]\,\rangle$ for p not occurring in A, B

THEN $A \equiv B$.

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Except for spoiling the whole equational story,
it does not sit too well with the labelling axiom . . .

The Nasty Trick Does Its Job

. . . but it's perfect for obtaining complexity results
for query equivalence problem
by using reductions to corresponding modal logics

Complexity Theorem

Theorem

- *Query equivalence of Core XPath($\rightarrow^+, \leftarrow^+$), Core XPath($\uparrow^+$), Core XPath($\mathsf{s}$) (for $\mathsf{s} \in \{\uparrow, \leftarrow, \rightarrow\}$) is coNP-complete.*
- *Query equivalence of Core XPath($\leftarrow^+, \leftarrow, \rightarrow^+, \rightarrow, \uparrow^+, \uparrow$) is PSPACE-complete.*

  Thus, the PSPACE upper bound applies to all its sublanguages.
- *Query equivalence of Core XPath($\downarrow$) and Core XPath($\downarrow^+$) is PSPACE-complete.*

  Thus, all extensions of this fragment are PSPACE-hard.
- *Query equivalence of Core XPath($\downarrow, \downarrow^+$) is EXPTIME-complete.*

  Thus, all extensions of this fragment are EXPTIME-hard.

Proofs

…by reductions to complexity results for modal logics like K, K4, Alt.1 and fragments of tense/temporal logic on linear and branching orders.

The most interesting one is for the second clause—somewhat tricky embedding into a logic of Sistla and Clarke.

What we have seen so far . . .

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- equational axiomatizations for path equivalences of all eight single axis fragments of Core XPath
- equational axiomatizations for node equivalences of full Core XPath 1.0
- non-orthodox axiomatization for path equivalences of full Core XPath 1.0
- computational complexity results for path equivalences in most meaningful sublanguages of Core XPath 1.0

What we have not seen so far . . .

- Definability and expressivity results (for finite sibling-ordered trees . . .)
- Results for fragments of XPath stronger than CoreXPath 1.0

From now on, I am going to use Balder Ten Cate's M4M 2007 slides

Expressive power

Possible yardsticks for expressive power on trees:

- First-order logic (FO), (cf. Codd completeness of SQL/RA)
- Monadic second-order logic (MSO)
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Decidable characterizations?

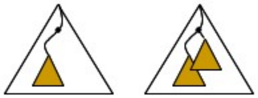
Expressive power (ct'd)

Descendant-only fragment

CoreXPath($\downarrow^*$) node expressions have the same expressive power as MSO formulas $\varphi(x)$ for which

(i) truth of $\varphi(x)$ at a node depends only on the subtree

(ii) $\varphi(x)$ does not distinguish children from descendants, i.e., the following operation preserves truth/falsity at the root:



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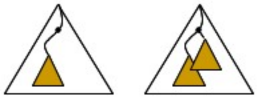

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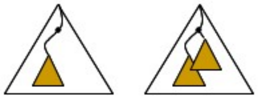

Easy proof from *De Jongh-Sambin fixed point theorem for GL* and *Janin-Walukiewicz theorem for μ-calculus*, see M4M proceedings paper.

Moreover, the proof is *effective*: it yields a decision procedure.

A Lost Exercise

Exercise

1. Prove that finite sibling-ordered trees are bisimiliar iff they are ismorphic
2. Does this result hold for arbitrary trees?

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Syntactic characterization of Core XPath (Marx-De Rijke 05)

Core XPath node expressions have the same expressive power as formulas $\phi(x)$ in the *two-variable fragment of* $FO[R_{\downarrow}, R_{\downarrow^*}, R_{\rightarrow}, R_{\rightarrow^*}]$.
There is a similar characterization for *path expressions*.

Brief recap

In summary...

- Core XPath is *reasonably expressive* yet *computationally attractive*.

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  1. *Regular XPath*: the extension of Core XPath with full *transitive closure*.

Brief recap

In summary…

- Core XPath is *reasonably expressive* yet *computationally attractive*.

- In the remainder of this talk, we consider two extensions:

  1. *Regular XPath*: the extension of Core XPath with full *transitive closure*.
  2. *Core XPath 2.0*: the navigational core of XPath 2.0, featuring *path intersection and complementation* and more.

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The extension of Core XPath with transitive closure is called *Regular XPath*.

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  - *path expressions*
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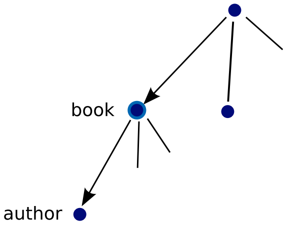
  - *node expressions*
    $\phi \ ::= \ p \mid \neg\phi \mid \phi \wedge \psi \mid \langle\alpha\rangle$

An example



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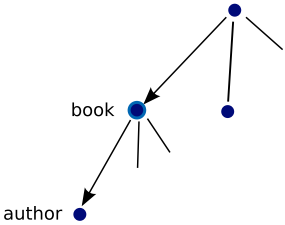

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In Regular XPath:

$$(\rightarrow [\neg \mathit{twoauthorbook}])^{*} / \rightarrow [\mathit{twoauthorbook}]$$

where *twoauthorbook* stands for
$\mathit{book} \wedge \langle \downarrow [\mathit{author}] / \rightarrow^{+} [\mathit{author}] \rangle$.

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- Let (α *while* ϕ) be shorthand for $(.[\phi]/\alpha)^*$.
- Let *root* be short for $\neg\langle\uparrow\rangle$.
  Let *leaf* be short for $\neg\langle\downarrow\rangle$.
  Let *first* be short for $\neg\langle\leftarrow\rangle$.
  Let *last* be short for $\neg\langle\rightarrow\rangle$.

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To see this, note that

- Let (α *while* ϕ) be shorthand for $(.[\phi]/\alpha)^*$.
- Let *root* be short for $\neg\langle\uparrow\rangle$.
  Let *leaf* be short for $\neg\langle\downarrow\rangle$.
  Let *first* be short for $\neg\langle\leftarrow\rangle$.
  Let *last* be short for $\neg\langle\rightarrow\rangle$.
- Let *suc* be shorthand for $\downarrow[\textit{first}] \cup .[\textit{leaf}]/(\uparrow \textit{ while last})/\rightarrow$
  (the successor in depth first left-to-right ordering).

Another example

The following can be expressed in Regular XPath:

"The tree has an even number of nodes"

To see this, note that

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- Let *suc* be shorthand for $\downarrow[\mathit{first}] \cup .[\mathit{leaf}]/(\uparrow \textit{ while last})/\rightarrow$ (the successor in depth first left-to-right ordering).
- Then $\Big\langle(\mathit{suc}/\mathit{suc})^*[\mathit{leaf}]/(\uparrow \textit{ while last})[\mathit{root}]\Big\rangle$ is true at the root iff the number of nodes is even.

One more example

- Consider game trees:
  - *leafs* are labeled by *Anne-wins* or *Bob-wins*
  - *inner nodes* are labeled by *Anne's-move* or *Bob's-move*

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- Consider game trees:
  - *leafs* are labeled by *Anne-wins* or *Bob-wins*
  - *inner nodes* are labeled by *Anne's-move* or *Bob's-move*
- Puzzle:
  Show that "*Anne has a winning strategy*" is expressible.

Expressive power of Regular XPath

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- We know that

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(after all, Regular XPath *has* a transitive closure operator!)

- We managed to prove this only if we extend Regular XPath with a "*within*" operator W:

$$T, n \models W\phi \quad \text{iff} \quad T_n, n \models \phi$$

(cf. temporal logics with forgettable past)

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- **Corollary:** Regular XPath(W) is closed under path intersection and complementation.

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As for complexity,

- Query evaluation can still be performed in *PTime* even for *Regular XPath(W)*.
- Query containment is still *ExpTime-complete* for *Regular XPath* but it is *2ExpTime-complete* for *Regular XPath(W)*

Core XPath 2.0

XPath 2.0 extends XPath 1.0 with many features, including the following new navigational operations:

- *Intersection* and *complementation* of path expressions.
  α `intersect` β and α `except` β

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- Core XPath 2.0 is the extension of Core XPath with these features.

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- The *variables and* `for`*-loops* make it possible to give a linear translation from first-order logic to Core XPath 2.0:

$$TR(\phi(x,y)) = \texttt{for \$x in ., \$y in } \top \texttt{ return \$y[}TR'(\phi)\texttt{]}$$

$$\begin{aligned}
TR'(x=y) &= \langle \top\texttt{[. is \$x} \wedge \texttt{. is \$y]}\rangle \\
TR'(R_{\downarrow}xy) &= \langle \top\texttt{[. is \$x} \wedge \langle\downarrow\texttt{[. is \$y]}\rangle\texttt{]}\rangle \\
TR'(R_{\downarrow^*}xy) &= \langle \top\texttt{[. is \$x} \wedge \langle\downarrow^*\texttt{[. is \$y]}\rangle\texttt{]}\rangle \\
TR'(R_{\rightarrow}xy) &= \langle \top\texttt{[. is \$x} \wedge \langle\rightarrow\texttt{[. is \$y]}\rangle\texttt{]}\rangle \\
TR'(R_{\rightarrow^*}xy) &= \langle \top\texttt{[. is \$x} \wedge \langle\rightarrow^*\texttt{[. is \$y]}\rangle\texttt{]}\rangle
\end{aligned}$$

$$\begin{aligned}
TR'(\phi \wedge \psi) &= TR'(\phi) \wedge TR'(\psi) \\
TR'(\neg\phi) &= \neg TR'(\phi) \\
TR'(\exists x.\phi) &= \texttt{for \$x in } \top \texttt{ return } TR'(\phi)
\end{aligned}$$

where $\top$ is shorthand for $\uparrow^* / \downarrow^*$ (the universal relation)

Expressivity and complexity

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- Is there anything interesting left to say about XPath 2.0?
- Sure! For example, *axiomatization*.
- We have two *complete axiomatizations* of path equivalence in Core XPath 2.0: one with and one without variables.

The case without variables

- Recall that, without variables, Core XPath 2.0 is essentially a version of *Relation Algebra* interpreted on *finite sibling ordered trees*.

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- One apparent problem: Relation Algebra has no node tests.
  However, these can easily be translated away:

$$\begin{array}{llcl}
\textit{Pred}1. & \alpha[\phi \wedge \psi] & \equiv & \alpha[\phi][\psi] \\
\textit{Pred}2. & \alpha[\phi \vee \psi] & \equiv & \alpha[\phi] \cup \alpha[\psi] \\
\textit{Pred}3. & \alpha[\neg\phi] & \equiv & \alpha - \alpha[\phi] \\
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- Besides these axioms, our axiomatization for variable free Core XPath 2.0 contains *two groups of axioms*:
  - General axioms of Boolean Algebra and Relation Algebra
  - Axioms describing (first-order) properties of trees.

Axioms of Boolean algebra

- BA1. $\alpha \cup (\beta \cup \gamma) \equiv (\alpha \cup \beta) \cup \gamma$
- BA2. $\alpha \cap (\beta \cap \gamma) \equiv (\alpha \cap \beta) \cap \gamma$
- BA3. $\alpha \cup \beta \equiv \beta \cup \alpha$
- BA4. $\alpha \cap \beta \equiv \beta \cap \alpha$
- BA5. $\alpha \cup (\beta \cap \gamma) \equiv (\alpha \cup \beta) \cap (\alpha \cup \gamma)$
- BA6. $\alpha \cap (\beta \cup \gamma) \equiv (\alpha \cap \beta) \cup (\alpha \cap \gamma)$
- BA7. $\alpha \cup (\alpha \cap \beta) \equiv \alpha$
- BA8. $\alpha \cap (\alpha \cup \beta) \equiv \alpha$
- BA9. $\alpha \cup (\top - \alpha) \equiv \top$
- BA10. $\alpha \cap (\top - \alpha) \equiv \bot$
- BA11. $\alpha \cap (\top - \beta) \equiv \alpha - \beta$

The axioms of Relation algebra

- RA1. $\alpha/(\beta/\gamma) \equiv (\alpha/\beta)/\gamma$
- RA2. $\alpha/. \equiv \alpha$
- RA3. $(\alpha \cup \beta)/\gamma \equiv \alpha/\gamma \cup \beta/\gamma$
- RA4. $(\alpha \cup \beta)^{\smile} \equiv \alpha^{\smile} \cup \beta^{\smile}$
- RA5. $(\alpha/\beta)^{\smile} \equiv \beta^{\smile}/\alpha^{\smile}$
- RA6. $(\alpha^{\smile})^{\smile} \equiv \alpha$
- RA7. $(\alpha/(\top - (\alpha^{\smile}/\beta)) \subseteq \top \ \texttt{except} \ \beta$

- To completely axiomatize relation algebra, normally, one needs to add also *Venema's Rule*:

  If X is a relation variable not occurring in α *and* $X - (((\top - .)/X/\top) \cup (\top/X/(\top - .))) \subseteq \alpha$ *then* $\alpha \equiv \top$*.*

  Fortunately, this rule turns out to be derivable in our case.

The axioms for finite sibling ordered trees

*Tr*1.	$\downarrow^+ / \downarrow^+$	$\subseteq$	$\downarrow^+$
*Tr*2.	$\downarrow^+ \cap \uparrow^+$	$\equiv$	$\bot$
*Tr*3*a*.	$\downarrow^+$	$\equiv$	$\downarrow \cup (\downarrow / \downarrow^+)$
*Tr*3*b*.	$\downarrow$	$\equiv$	$\downarrow^+ - (\downarrow^+ / \downarrow^+)$
*Tr*4.	$.[\langle\uparrow\rangle]$	$\equiv$	$.[\uparrow^+ [\neg\langle\uparrow\rangle]]$
*Tr*5.	$\downarrow^+ / \uparrow^+$	$\equiv$	$\downarrow^+ \cup .[\langle\downarrow\rangle] \cup .[\langle\downarrow\rangle] / \uparrow^+$
*Tr*6.	$\rightarrow^+ / \rightarrow^+$	$\subseteq$	$\rightarrow^+$
*Tr*7.	$\rightarrow^+ \cap \leftarrow^+$	$\equiv$	$\bot$
*Tr*8*a*.	$\rightarrow^+$	$\equiv$	$\rightarrow \cup (\rightarrow / \rightarrow^+)$
*Tr*8*b*.	$\rightarrow$	$\equiv$	$\rightarrow^+ - (\rightarrow^+ / \rightarrow^+)$
*Tr*9.	$.[\leftarrow]$	$\equiv$	$.[\langle\leftarrow^+ [\neg\langle\leftarrow\rangle]\rangle]$
*Tr*10.	$\rightarrow^+ \cup \leftarrow^+$	$\equiv$	$(\uparrow / \downarrow) - .$
*Tr*11.	$. \cup \uparrow^+ \cup \downarrow^+ \cup$ $(\uparrow^* / \rightarrow^+ / \downarrow^*) \cup (\uparrow^* / \leftarrow^+ / \downarrow^*)$	$\equiv$	$\top$
Ind.	$\top[\langle\alpha\rangle]$	$\equiv$	$\top[\langle\alpha - (\alpha / <<)\rangle]$

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  Expressivity: FO^2
  Query evaluation: PTime
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- **Core XPath**: the navigational core of XPath 1.0
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  Expressivity: same as $FO(TC)$
  Query evaluation: PTime
  Query equivalence: 2ExpTime-complete.

- **Core XPath 2.0**: the navigational core of XPath 2.0
  Expressivity: same as FO.
  Query evaluation: PSpace-complete
  Query equivalence: non-elementary hard.

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