Queries, Modalities, Relations, Trees, XPath
Lecture VI
Harvest: Core XPath 1.0
as a Modal Logic for Trees
Axiomatizations and Complexity

Tadeusz Litak

Department of Computer Science
University of Leicester

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Before we begin . . .

I am sometimes asked by students (not researchers, as they know the answer)

why theory is needed?
is it possible to just do applications
without any theory?

The answer

was provided by

Charles-Louis de Secondat,
baron de La Bréde et de Montesquieu,
one of the greatest European philosophers of law
(XVIIIth century)

An Idea of Despotic Power

When the savages of Louisiana are desirous of fruit,

they cut the tree to the root, and gather the fruit.

This is an emblem of despotic government.

Trying to learn "applications" without necessary background

means **cutting the tree of learning**

In the long run, you are not going to have any fruits

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- how closely things are connected
- how many tools and techniques from the past you can reuse

FROM NOW ON, THE LECTURE IS MEANT TO ILLUSTRATE THIS

To use José's terminology, the contents belong

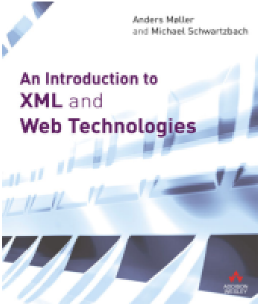
to Web Services Architecture

and more specifically, of course,

to XML Technologies

XML and Web Technologies

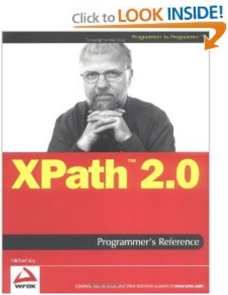
A good overall reference:



Webpage for the book: *http://www.brics.dk/ixwt/*

XPath

Most detailed reference on XPath
(except for W3C specification itself):



XML and Semi-structured Data

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eXtensible Markup Language

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  - XHTML . . .

Example Document

No XML talk can do without its own example document:

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```
<?xml version='1.0' encoding='UTF-8'?>
<talk date='23-Jul-2010'>
  <speaker uni='Leicester'>T. Litak</speaker>
  <title>
    <i>Core XPath 1.0</i>as a Modal Logic
  </title>
  <location>
    <i>JXNU</i><b>Nanchang</b>
  </location>
</talk>
```

(no DTD given, but you can easily come up with one)

What we'll see through our dim glasses

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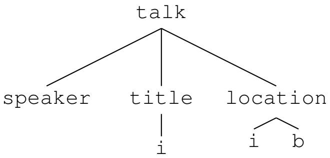
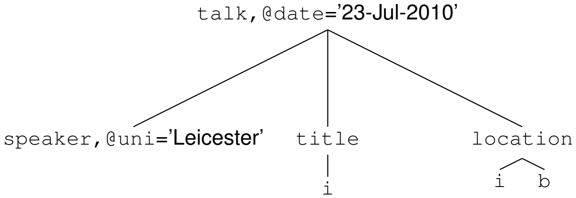
Either this . . .



(we cannot even see attributes, each node is labelled with a single label: its name)

What we'll see through our dim glasses

or that ...



(attribute-value pairs are additional labels)

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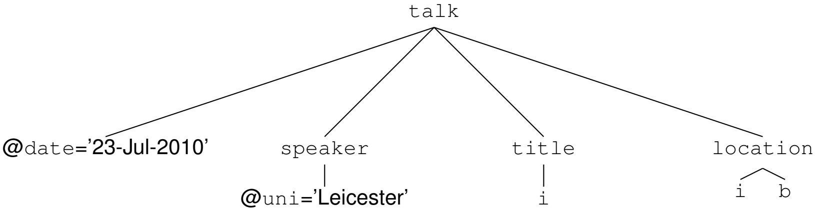

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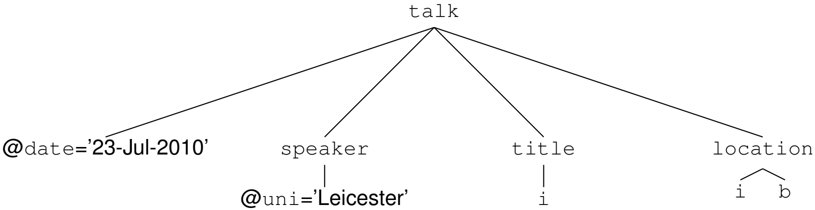

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At any rate, we are too blind to see actual text content

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- *Uses a compact, non-XML syntax to facilitate use of XPath within URIs and XML attribute values*
- *Operates on the abstract, logical structure of an XML document, rather than its surface syntax*

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- Specification of XPath 3.0: $\pm$ 270 pages? (Balder's extrapolation)

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An additional advantage of such a simple language:

data model discrepancies between XPath 1.0 and 2.0
no longer relevant

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`...self::*`, pexpr/pexpr, pexpr | pexpr, pexpr[nexpr]
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`self::`*p* , pexpr, `not`(nexpr), nexpr `or` nexpr

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We also consider single axis fragments of CoreXPath—notation CoreXPath(a) for a fixed axis a

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finite sibling-ordered node labelled trees:
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- $V : \Sigma \rightarrow 2^N$ (or just $V : N \rightarrow \Sigma$ if unique labelling assumed)

Semantics of Core XPath II

$$
\begin{array}{ll}
\text{pexpr}: & \text{pairs (context node, reachable node)—subsets of } N^2 \\
[\![\mathsf{s}]\!]^T & = R_{\mathsf{s}} \\
[\![\mathsf{s}^+]\!]^T & = \text{the transitive closure of } R_{\mathsf{s}} \\
[\![\cdot]\!]^T & = \text{the identity relation on } N \\
[\![A/B]\!]^T & = \text{composition of } [\![A]\!]^T \text{ and } [\![B]\!]^T \\
[\![A \cup B]\!]^T & = \text{union of } [\![A]\!]^T \text{ and } [\![B]\!]^T \\
[\![A[\phi]]\!]^T & = \{(n, m) \in [\![A]\!]^T \mid m \in [\![\phi]\!]^T\}
\end{array}
$$

$$
\begin{array}{ll}
& \text{nexpr}: \text{ subsets of } N \\
[\![p]\!]^T & = \{n \in N \mid n \in V(p)\} \\
[\![\phi \wedge \psi]\!]^T & = [\![\phi]\!]^T \cap [\![\psi]\!]^T \\
[\![\neg\phi]\!]^T & = N \setminus [\![\phi]\!]^T \\
[\![\langle A \rangle]\!]^T & = \text{domain of } [\![A]\!]^T = \{n \mid (n, m) \in [\![A]\!]^T\}
\end{array}
$$

Remember what we've seen yesterday?

A (slightly modified) diagram of Johan Van Benthem:

$\underline{\mathfrak{W}}$				$\underline{\mathfrak{W}}^2$
unary properties	$\longrightarrow$	*modes*	$\longrightarrow$	binary relations
of states	$\longleftarrow$	*projections*	$\longleftarrow$	between states
propositional operators				program operators
ML				DRA/TRA

Examples of modes:

$$?X := \{\langle x, x\rangle \mid x \in X\} \qquad \text{(testing)}$$

$$!X := \{\langle w, x\rangle \mid w \in \underline{\mathfrak{W}}, x \in X\} \qquad \text{(realizing)}$$

Examples of projections:

$$\langle R\rangle := \{w \in \underline{\mathfrak{W}} \mid \exists v \in \underline{\mathfrak{W}}. wR^{\mathfrak{W}}v\} \qquad \text{(domain)}$$

$$\pi^{-1}(R) := \{w \in \underline{\mathfrak{W}} \mid \exists v \in \underline{\mathfrak{W}}. vR^{\mathfrak{W}}w\} \qquad \text{(codomain)}$$

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NOTE THAT:

$$\langle R\rangle = \sim\sim R$$

$$= R/R^{\smile} \cap \cdot$$

$$\Delta(R) = R \cap \cdot$$

Comments for logicians

- Note we do not allow transitive closure of arbitrary path expressions (allowed in non-standard extensions like Regular XPath)
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Syntax of Short Core XPath:

$s ::= \downarrow, \uparrow, \leftarrow, \rightarrow$
$a ::= s \mid s^{+}$
$exp ::= \cdot \mid a \mid exp/exp \mid exp \cup exp \mid ?p \mid \sim exp \qquad (p \in \Sigma)$

Definition of a single axis fragment remains the same

Semantics of Short Core XPath

$$
\begin{array}{ll}
\text{pexpr}: & \text{pairs (context node, reachable node)—subsets of } N^2 \\
[\![\mathsf{s}]\!]^T & = R_{\mathsf{s}} \\
[\![\mathsf{s}^+]\!]^T & = \text{the transitive closure of } R_{\mathsf{s}} \\
[\![\cdot]\!]^T & = \text{the identity relation on } N \\
[\![A/B]\!]^T & = \text{composition of } [\![A]\!]^T \text{ and } [\![B]\!]^T \\
[\![A \cup B]\!]^T & = \text{union of } [\![A]\!]^T \text{ and } [\![B]\!]^T \\
[\![A[\phi]]\!]^T & = \{(n,m) \in [\![A]\!]^T \mid m \in [\![\phi]\!]^T\} \\
 & \qquad \text{nexpr}: \ \text{subsets of } N \\
[\![p]\!]^T & = \{n \in N \mid n \in V(p)\} \\
[\![\phi \wedge \psi]\!]^T & = [\![\phi]\!]^T \cap [\![\psi]\!]^T \\
[\![\neg\phi]\!]^T & = N \setminus [\![\phi]\!]^T \\
[\![\langle A \rangle]\!]^T & = \text{domain of } [\![A]\!]^T = \{n \mid (n,m) \in [\![A]\!]^T\}
\end{array}
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$$
\begin{aligned}
[\![?p]\!]^T &= \{(n,n) \in N^2 \mid n \in V(p)\}\\
[\![\sim A]\!]^T &= \{(n,n) \in N^2 \mid \forall m.(n,m) \notin [\![A]\!]^T\}
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$$

Back-and-forth Between Core XPath and SCX

One direction is easy:
$[\![\sim A]\!]^T = [\![\cdot[\neg\langle A \rangle]]\!]^T$

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t(p) &= ?p \\
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$$[\![A]\!]^T = [\![t(A)]\!]^T \qquad \text{for all } A \in \text{pexpr}$$

$$[\![\cdot[\phi]]\!]^T = [\![t(\phi)]\!]^T \qquad \text{for all } \phi \in \text{nexpr}$$

When Two Queries Are Equivalent?

Definition

Let P and Q be either

- both path expressions or
- both node expressions

We say P and Q are equivalent ($P \equiv Q$)
if for any document $[\![P]\!]^T = [\![Q]\!]^T$

Which expressions are equivalent?

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- One last try: how about
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Evaluation times of two equivalent queries
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(completeness problem)
. . . you took care of all (possibly) relevant ones?
there might be classes of equivalences you never thought of!

Definition (Complete Axiomatization)

A complete axiomatization of a given XPath fragment:

A set of

- finitely many valid equivalence schemes
- finitely many validity preserving inference rules

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One of reasons why we consider Core XPath only:

the whole XPath would be too big to allow an axiomatization

How does a complete axiomatization help?

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- Hopefully, it should also yield better rewrite strategies

Logic—Algebra—Query Languages

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logic:	algebras:	databases:

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Clearly, these rules preserve validity.

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Before we proceed, you may have two questions:

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- . . . the avalanche of results it triggered off? :)
  Theory of varieties developed since the 1930's: semigroups and groups, semirings, semilattices, lattices and residuated lattices, boolean algebras, abstract relation and cylindric algebras . . .

Q1 cntnd: But What Use Are They For Us?

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Almost all axiomatizations presented today will be orthodox
(you're going to see one exception at the end of the talk and dislike it)

Q2: Anything Special about XPath?

Question

How about complete axiomatizations for SQL-like languages?

After all, there has been nothing XML specific to what I said . . .

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Some database theorists got into problems not knowing about it . . .

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Some database theorists got into problems not knowing about it . . .
It does not mean you cannot find interesting axiomatizable fragments—they are rather small though

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Short Answer

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Basic Axioms I: Idempotent Semirings

ISAx1	$(A \cup B) \cup C$	$\equiv$	$A \cup (B \cup C)$
ISAx2	$A \cup B$	$\equiv$	$B \cup A$
ISAx3	$A \cup A$	$\equiv$	A
ISAx4	$A/(B/C)$	$\equiv$	$(A/B)/C$
ISAx5	$\cdot/A$	$\equiv$	A
	$A/\cdot$	$\equiv$	A
ISAx6	$A/(B \cup C)$	$\equiv$	$A/B \cup A/C$
	$(A \cup B)/C$	$\equiv$	$A/C \cup B/C$
ISAx7	$\bot$	$\subseteq$	A

Distributive lattices, Kleene algebras, Tarski's relation algebras: they all have idempotent semiring reducts.
Idempotency is the axiom ISAx3.
$\bot$ abbreviates $\cdot\,[\neg\langle\cdot\rangle]$

Basic Axioms II: Predicate Axioms

PrAx1	$A[\neg\langle B\rangle]/B$	$\equiv$	$\bot$
PrAx2	$A[\phi \vee \psi]$	$\equiv$	$A[\phi] \cup A[\psi]$
PrAx3	$(A/B)[\phi]$	$\equiv$	$A/B[\phi]$
PrAx4	$\cdot[\langle\cdot\rangle]$	$\equiv$	$\cdot$

In Tarski's relation algebras and XPath 2.0,
predicates can be defined away
Note that PrAx3 would not be valid
if we allowed unrestricted positional predicates

Basic Axioms III: Node Axioms

NdAx1	ϕ	$\equiv$	$\neg(\neg\phi \vee \psi) \vee \neg(\neg\phi \vee \neg\psi)$
NdAx2	$\langle A \cup B \rangle$	$\equiv$	$\langle A \rangle \vee \langle B \rangle$
NdAx3	$\langle A/B \rangle$	$\equiv$	$\langle A[\langle B \rangle] \rangle$
NdAx4	$\langle \cdot [\phi] \rangle$	$\equiv$	ϕ

Note how little was needed to ensure booleanity!
(by Huntington's result from the 1930's)
And NdAx2–NdAx4 just mimick PrAx2—PrAx4
(redundancy: price to pay for two-sorted signature)

Axioms in one-sorted signature

Recall all the two-sorted axioms for predicates and expressions:

PrAx1	$A\,[\neg\langle B\rangle]\,/B$	$\equiv$	$\bot$
PrAx2	$A\,[\phi \vee \psi]$	$\equiv$	$A\,[\phi] \cup A\,[\psi]$
PrAx3	$(A/B)\,[\phi]$	$\equiv$	$A/B\,[\phi]$
PrAx4	$\cdot\,[\langle\cdot\rangle]$	$\equiv$	$\cdot$
NdAx1	ϕ	$\equiv$	$\neg(\neg\phi \vee \psi) \vee \neg(\neg\phi \vee \neg\psi)$
NdAx2	$\langle A \cup B\rangle$	$\equiv$	$\langle A\rangle \vee \langle B\rangle$
NdAx3	$\langle A/B\rangle$	$\equiv$	$\langle A\,[\langle B\rangle]\,\rangle$
NdAx4	$\langle\cdot\,[\phi]\,\rangle$	$\equiv$	ϕ

Axioms in one-sorted signature

Here is a one-sorted axiomatization for $\sim$ over idempotent semi-ring axioms found by Hollenberg:

$$
\begin{array}{lcl}
\sim A/A & \equiv & \bot \\
\sim\sim A/A & \equiv & A \\
\sim(A/B)/A & \equiv & (\sim(A/B)/A)/\sim B \\
\sim(A \cup B) & \equiv & \sim A/\sim B \\
\sim A \cup \sim B & \equiv & \sim\sim(\sim A \cup \sim B)
\end{array}
$$

We need to add one more axiom for tests:

$$?p \equiv \sim\sim ?p$$

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Core XPath($\downarrow$), the child-axis-only fragment!

Theorem

The axioms presented so far are complete for all valid equivalences of Core XPath($\downarrow$).

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Theorem

The axioms presented so far are complete for all valid equivalences of Core XPath($\downarrow$).

In order to find more interesting equivalences,
we have to move to other fragments

Axioms for Linear Axes

The non-transitive case:

$$\text{LinAx1} \quad \mathsf{s}[\neg\phi] \quad \equiv \quad \cdot[\neg\langle \mathsf{s}[\phi]\rangle]/\mathsf{s} \quad \text{for } \mathsf{s} \in \{\rightarrow, \leftarrow, \uparrow\}$$

This forces functionality of the corresponding axis

Axioms for Transitive Axes

One for node expressions, one for path expressions:

TransAx1	$\langle s^+[\phi]\rangle$	$\equiv$	$\langle s^+[\phi \wedge \neg\langle s^+[\phi]\rangle]\rangle$
TransAx2	s^+	$\equiv$	$s^+ \cup s^+/s^+$

The first one is called the Löb axiom and forces well-foundedness
Don't get modal logicians started on it—
people wrote books about this formula

In particular, all the consequences of TransAx2 for *node expressions*
can be already derived from TransAx1
I can neither prove nor disprove that for *path expressions*
TransAx2 is (ir-)redundant

Finally, Axes which Are Both Transitive and Linear

LinAx2 $\cdot[\langle s^+[\phi]\rangle]/s^+ \equiv s^+[\phi] \cup s^+[\phi]/s^+ \cup s^+[\langle s^+[\phi]\rangle]$
for $s \in \{\rightarrow, \leftarrow, \uparrow\}$

together with transitivity axioms

This forces the corresponding axis is a linear order

Single Axis Completeness Result

Theorem

- *Base axioms are complete for Core XPath($\downarrow$)*
- *Base axioms with LinAx1 are complete for other intransitive single axis fragments*
- *Base axioms with TransAx1 and TransAx2 are complete for Core XPath($\downarrow^+$)*
- *Base axioms with TransAx1, TransAx2 and LinAx2 are complete for other transitive single axis fragments*

A Few Words About Proofs

- First, rewrite node expressions to simple node expressions:

$$\text{siNode} ::= \langle\cdot\rangle \mid p \mid \langle \text{a}\,[\text{siNode}]\,\rangle \mid \neg\text{siNode} \mid \text{siNode} \vee \text{siNode}$$

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  They are isomorphic variants of modal formulas
- Using normal form theorems for modal logic, we provide a completeness proof for node expressions

A Few Words About Proofs cntd.

- Then we rewrite all path expressions as sums of sum-free expressions of the form

$$S = \cdot\,[\beta_1]\,/\mathsf{a}\,[\beta_2]\,/\ldots/\mathsf{a}\,[\beta_\ell]\,,$$

(all β_i are normal forms of

- the same nesting degree in case of transitive axes
- strictly decreasing degree for intransitive axes)

In case of linear axes, we can even guarantee that every formula is witnessed further down the chain

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  - there is a countermodel for containment

Aside: the issue of labels

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This axiom itself is not substitution-invariant,
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But as our proofs used only Birkhoff's rules
they are quite flexible and adding this axiom does not hurt

Starting from the Other End

Instead of beginning with single axes
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LET'S GO FOR THE WHOLE CORE XPATH!

Axiom For Axes Dependencies

$$\text{TreeAx1}\begin{cases} s^{+}/s \cup s \equiv s^{+} \\ s/s^{+} \cup s \equiv s^{+} \end{cases}$$

$$\text{TreeAx2}\quad s\,[\phi]\,/s^{\smile} \equiv \cdot\,[\langle s\,[\phi]\,\rangle]\ (\text{ for s distinct than } \uparrow)$$

$$\text{TreeAx3}\quad \uparrow[\phi]\,/\!\downarrow \equiv (\leftarrow^{+} \cup \rightarrow^{+} \cup \cdot)\,[\langle\uparrow[\phi]\,\rangle]$$

$$\text{TreeAx4}\begin{cases} \leftarrow^{+} \equiv \leftarrow^{+}[\langle\uparrow\rangle] \\ \rightarrow^{+} \equiv \rightarrow^{+}[\langle\uparrow\rangle] \end{cases}$$

TreeAx1 says: s^{+} is a transitive closure of s
TreeAx2 says non-child axes are functional
and describes their converse
TreeAx3 forces $\uparrow$ is the converse of (non-functional) $\downarrow$
with TreeAx4, it also describes how horizontal and vertical axes interplay

Theorem

The axioms presented so far are complete for Core XPath node expressions

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Proof.

By reduction to simple node expressions
and derivation of all axioms of modal logic of finite trees
by Blackburn, Meyer-Viol, de Rijke □

(boolean axioms)

$$\langle s[\neg\langle\cdot\rangle]\rangle \equiv \neg\langle\cdot\rangle$$
$$\langle s[\phi \vee \psi]\rangle \equiv \langle s[\phi]\rangle \vee \langle s[\psi]\rangle$$
$$\phi \leq \neg\langle s[\neg\langle s^{\smile}[\phi]\rangle]\rangle$$
$$\langle s[\neg\phi]\rangle \wedge \langle s[\phi]\rangle \equiv \neg\langle\cdot\rangle (\text{ for s distinct than } \uparrow)$$
$$\langle s[\phi]\rangle \vee \langle s[\langle s^{+}[\phi]\rangle]\rangle \equiv \langle s^{+}[\phi]\rangle$$
$$\neg\langle s[\phi]\rangle \wedge \langle s^{+}[\phi]\rangle \leq \langle s^{+}[\neg\phi \wedge \langle s[\phi]\rangle]\rangle$$
$$\langle s[\langle\cdot\rangle]\rangle \leq \langle s^{+}[\neg\langle s[\langle\cdot\rangle]\rangle]\rangle$$

TransAx1 for $\downarrow^{+}$ and $\rightarrow^{+}$

$$\langle\downarrow[\neg\langle\leftarrow\rangle \wedge \neg\langle\rightarrow^{*}[\phi]\rangle]\rangle \leq \neg\langle\downarrow[\phi]\rangle$$
$$\langle\downarrow[\phi]\rangle \leq \langle\downarrow[\neg\langle\leftarrow\rangle]\rangle \wedge \langle\downarrow[\neg\langle\rightarrow\rangle]\rangle$$
$$\neg\langle\uparrow\rangle \leq \neg\langle\leftarrow\rangle \wedge \neg\langle\rightarrow\rangle$$

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Add the separability rule:

(Sep) IF $\langle A[p] \rangle \equiv \langle B[p] \rangle$ for p not occurring in A, B

THEN $A \equiv B$.

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THEN $A \equiv B$.

Except for spoiling the whole equational story,
it does not sit too well with the labelling axiom ...

The Nasty Trick Does Its Job

...but it's perfect for obtaining complexity results
for query equivalence problem
by using reductions to corresponding modal logics

Complexity Theorem

Theorem

- *Query equivalence of Core XPath($\rightarrow^+, \leftarrow^+$), Core XPath($\uparrow^+$), Core XPath(s) (for s $\in \{\uparrow, \leftarrow, \rightarrow\}$) is coNP-complete.*
- *Query equivalence of Core XPath($\leftarrow^+, \leftarrow, \rightarrow^+, \rightarrow, \uparrow^+, \uparrow$) is PSPACE-complete.*

  Thus, the PSPACE upper bound applies to all its sublanguages.
- *Query equivalence of Core XPath($\downarrow$) and Core XPath($\downarrow^+$) is PSPACE-complete.*

  Thus, all extensions of this fragment are PSPACE-hard.
- *Query equivalence of Core XPath($\downarrow, \downarrow^+$) is EXPTIME-complete.*

  Thus, all extensions of this fragment are EXPTIME-hard.

Proofs

…by reductions to complexity results for modal logics like K, K4, Alt.1 and fragments of tense/temporal logic on linear and branching orders.

The most interesting one is for the second clause—somewhat tricky embedding into a logic of Sistla and Clarke.

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We have seen:

- equational axiomatizations for path equivalences of all eight single axis fragments of Core XPath
- equational axiomatizations for node equivalences of full Core XPath 1.0
- non-orthodox axiomatization for path equivalences of full Core XPath 1.0
- computational complexity results for path equivalences in most meaningful sublanguages of Core XPath 1.0

What we have not seen so far . . .

- Definability and expressivity results (for finite sibling-ordered trees . . .)
- Results for fragments of XPath stronger than CoreXPath 1.0

Both are discussed in Balder's M4M slides