Introducing open games

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Joint work with
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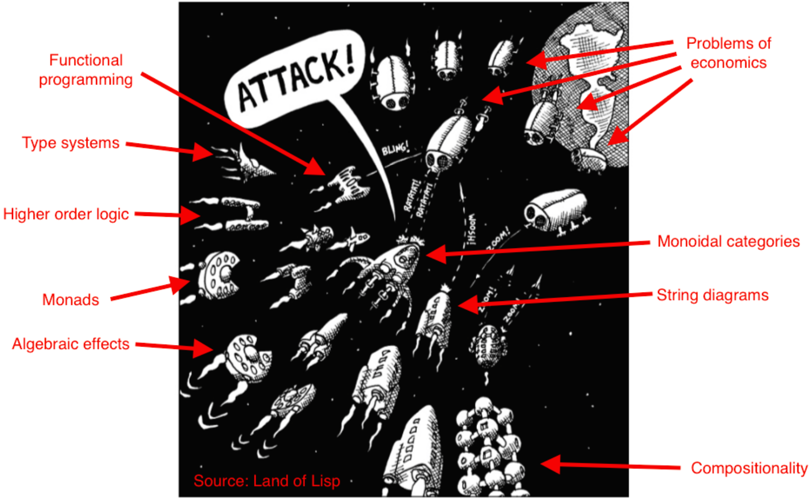
Motivation: powerful machinery vs. hard problems



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- "Local view" of rationality: players act to maximise payoff
- "Global view": equilibrium strategies

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Nash's theorem generalises this situation

Example: the $$$ auction

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- Ordinary games do not compose/scale

The fundamental headache of social science

Beliefs have causal effects

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- Objects can be composed without knowing how they are defined
- Consequence: can easily change one component in a large structure
- All reasoning is by structural induction on composition

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- Economic systems

The compositionality hypothesis

If a theory is compositional, then reasoning with it is scalable

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Hypothesis

Compositionality is the only way to be scalable

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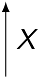
More generally:

- Leaky abstractions
- Emergent behaviour

Compositionality via symmetric monoidal categories (1)

Objects (aka. interfaces, types, systems) X



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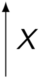

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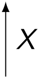

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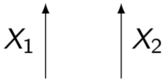
Compound object $X_1 \otimes X_2$



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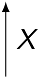

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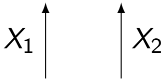
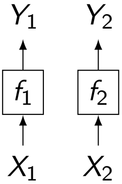
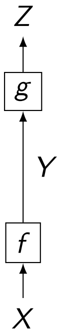


Monoidal product (aka. tensor product, simultaneous/spatial composition)
$f_1 \otimes f_2 : X_1 \otimes X_2 \to Y_1 \otimes Y_2$



Compositionality via symmetric monoidal categories (2)

Categorical composition (aka. sequential/temporal composition) $g \circ f : X \to Z$



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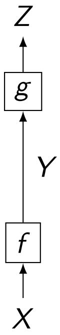
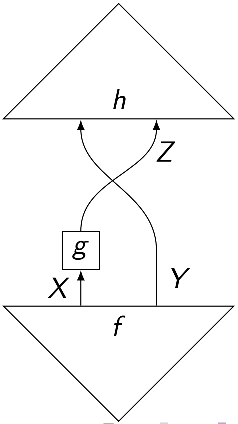


$f : I \to X \otimes Y$
$g : X \to Z$
$h : X \otimes Z \to I$
$h \circ \sigma_{Z,X} \circ (g \otimes Y) \circ f : I \to I$



Open games

A closed game consists of:

- A set Σ of strategy profiles
- A best response function $\mathbf{B} : \Sigma \to \mathcal{P}(\Sigma)$

An open game

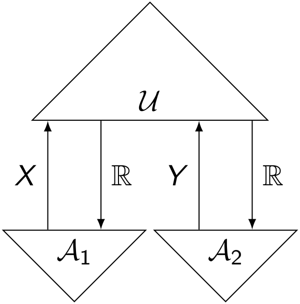
$$\mathcal{G} : (X, S) \to (Y, R)$$

consists of:

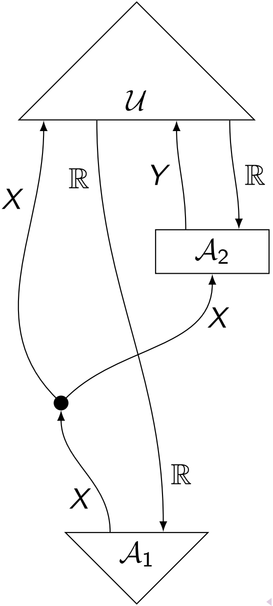
- A set Σ of strategy profiles
- A play function $\mathbf{P}_{\mathcal{G}} : \Sigma \times X \to Y$
- A coplay function $\mathbf{C}_{\mathcal{G}} : \Sigma \times X \times R \to S$
- A best response function

$$\mathbf{B}_{\mathcal{G}} : X \times (Y \to R) \times \Sigma \to \mathcal{P}(\Sigma)$$

Bimatrix game



Perfect information game



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